

## Post-cracking Behaviour and Fracture Energy of Synthetic Fibre Reinforced Concrete

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The paper reports the results of experimental programme focused on the effect of various synthetic fibres on fracture properties and ductility of concrete. The fracture energy was assessed on beams with initial notches in three-point bend test. The incorporation of synthetic fibres had a slight effect on mechanical properties of concrete but, at the same time, it had a significant influence on the fracture energy by modification of post-cracking behaviour of concrete. It was found that the modern synthetic fibres might be able to impart significant toughness and ductility to concrete. However, the beneficial effect of fibres depends on their length and flexibility. The analysis of load-deflection curves obtained made it possible to fit the simple function, describing the post-peak behaviour of fibre reinforced concrete, which can be useful for the calculation of  $G_F$  value.

*Keywords:* cement concrete, polypropylene fibre, post-peak behaviour, fracture energy.

### 1. INTRODUCTION

Over the past years, fibre reinforced concrete (FRC) has been widely used in different structural and non-structural applications such as pavements, floors, overlays, industrial slabs, and shortcorte lining etc. where the major concern is toughness and first-crack strength in flexure [1–3]. Depending on the distribution and orientation of fibres in cement matrix, the addition of fibres makes the cementitious material more isotropic and transforms it from a brittle to a quasi-brittle material. The real benefits of adding fibres to concrete become evident at the stage of post-cracking. Before that, the most of fibre types have a limited effect on concrete mechanical properties [4–6].

However, the full potential of fibre reinforced concrete is still not fully exploited in practice. This is mainly due to a lack of specific rules of fibre reinforced concrete in building codes. The existing rules for conventional concrete can hardly be adopted for fibre reinforced concrete that is markedly non linear since fibres start working after cracking of the concrete matrix [7].

The fracture mechanics, as one of the most significant field of science, is widely used to analyze the material behaviour in structure [8–10]. The fracture energy is a fundamental parameter of cohesive crack model, representing cracking resistance and fracture toughness of quasi-brittle materials e.g. concrete [9, 11]. The model is able to capture the essential features of a progressively fracturing surface and its evolution until the failure. The energy dissipation for crack propagation can be completely characterized by the cohesive stress-separation relationship [8]. It characterizes the softening response of a crack that could develop anywhere in a concrete structure. A unique load-displacement curve is very often used to quantify the value of energy [9]. The choice of descending softening function (or cohesive law) influences the prediction of the

structural response and the local fracture behaviour. Some of fitted functions were discussed in [11–14]. Fracture energy has been implemented in the mathematical models of various finite-element-programmes for non-linear modelling. Nevertheless, specific data concerning this material parameter that would provide advancement of the material models used and subsequently the calculated structural behaviour are hardly available [15].

The most of experiments relate to concrete reinforced by steel fibres and very few present the research on composites with other kinds of fibres i.e. synthetic fibres, which can present the series of advantages. With low modulus of elasticity, high strength, excellent ductility, excellent durability and low price, synthetic fibre is often used in cement-based materials to improve the ductility and fracture properties of the matrix [16–18]. Wang et al. [19] studied the effect of the polypropylene fiber upon the stress intensity factor, fracture energy and critical crack mouth opening displacement of polypropylene fiber reinforced high-strength concrete; the results indicate that the addition of polypropylene fibre increases the fracture energy and critical crack mouth opening displacement of high-strength concrete. The results obtained by Zhang et al. [20] indicate that silica fume and short polypropylene fiber both have great effect on the fracture parameters and relational curves of the three-point bending beam specimen.

In addition, in the recent years, important efforts have been devoted to develop new types of synthetic fibres [16]. There is little information available in literature about the fracture properties of modified polypropylene macro fibre reinforced composites. Therefore, the experimental study was conducted on a post-peak behaviour and fracture energy of concrete with macro fibres.

Model Code 2010 [21], RILEM TC 162 [22] and ACI 544 [1] provide the guidelines to designers of structures made of concrete with fibers. However, designers hardly accept the volunteering guidelines or research results available in scientific papers. For this reason, further

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research on fibre reinforced concrete is still needed to confirm its advantages, particularly the flexural toughness and fracture energy which are used for design purposes. The tensile test is of limited practical value because conditions of pure tension arise rarely in real components.

The aim of the research work was the analysis of the effect of various kinds of polypropylene fibres on the fracture energy  $G_F$  and post-cracking behaviour of fine grained concrete. In order to determine the value of fracture energy three-point bend tests were performed for a notched beams. On the basis of load-deflection curves analysis, the dependency composed of power function and linear function was proposed to describe the post-peak respons of concrete with synthetic fibre.

## 2. EXPERIMENTAL DETAILS

### 2.1. Materials and specimens preparation

The synthetic fibres of different surface finish were used. Two of them were smooth surface, straight fibres, made of the mix of polypropylene and polyethylene. They were characterized by the same mechanical properties but different length: type A (50 mm) and type B (25 mm). The third type (C) of dispersed reinforcement was structural, extruded fibre with the length of 50 mm, made of modified polypropylene. The type C fibre surface was crypsinated to improve the initial dispersion within the concrete. The treated surface should create a multi-directional bond between the fibre and the cement matrix. The fibres used were presented in Fig. 1 and the geometry and properties of fibres were given in Table 1. The fibres were added at three volume fractions 0.3 %, 0.6 % and 0.9 %, which range the dosage suggested by the producer. The fibres were introduced as a replacement of adequate portion of aggregate by volume.

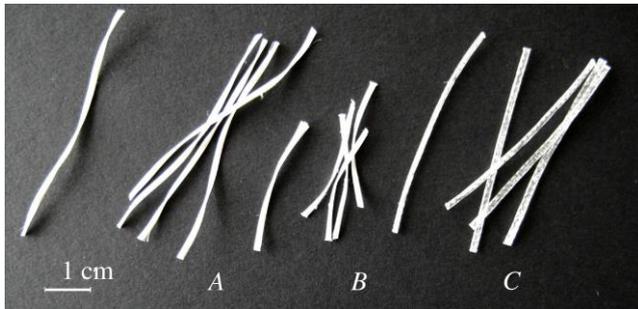


Fig. 1. Fibres used for tests

Table 1. Properties of fibres used

Property	CHRYSO Fibre S50 (A)	CHRYSO Fibre S25 (B)	CHRYSO Structural (C)
Fibre shape	straight	straight	crimped
Length, mm	50	25	50
Diameter/Section, mm	0.05	0.05	0.8 × 1.4
Tensile strength, MPa	600	600	550
Elastic modulus, GPa	5	5	6
Density, kg/m <sup>3</sup>	920	920	910

The effect of fibers on fracture properties was refereed to the results obtained for reference concrete without

fibres. Table 2 gives the mix proportions for reference concrete.

Table 2. Mix proportions of the reference concrete

Component	Dosage, kg/m <sup>3</sup>
Cement CEM I 42.5 R	400
Aggregate 0–2 mm	1245
Aggregate 2–4 mm	720
Water	160
Superplasticizer	3.2

The maximum size of aggregate was limited to reduce its influence on fracture properties and to provide the homogenous fibre distribution in concrete [1].

The modified polycarboxylate based super-plasticizer (CHRYSO Fluid Premium 380) was used to minimize fibre clumping and enhance fiber dispersion in concrete.

The dry aggregate was mixed with fibres followed by cement. The materials were dry mixed for 2 min before adding the water with super-plasticizer. Mixing continued for a further 4 min. The time of mixing was considered sufficient for the proper dispersion of the fibres in the mix without causing a “balling” effect.

For each fibre-dosage combination four notched beams of size 100 × 100 × 400 mm were used for three-point bending test. The initial saw-cut notch with a depth equal to 30 mm and width of 3 mm was located in the mid-span place. The elongated U-notches ( $a_0/d = 0.30$ ) were sawn under wet conditions one day before the test. Moreover, three beams (100 × 100 × 400 mm) for flexural strenght were also cast and three cubes (100 × 100 × 100 mm) for the compressive strength were cut from beam specimens.

The specimens were vibrated in moulds and than stored under polyethylene cover for one day. After demoulding all specimens were cured in water at the temperature of  $20 \pm 2$  °C till they were tested.

### 2.2. Test procedures

The fracture energy was assessed in three-point bend test on beams with initial notches according to procedure described in [23]. The geometry of specimen and the way of load were presented in Fig. 2.

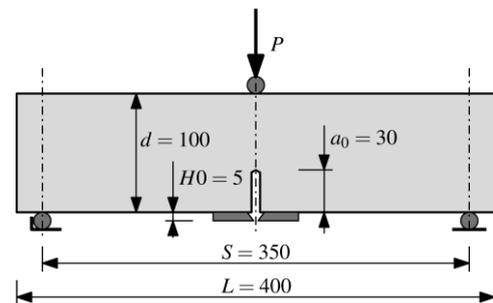


Fig. 2. Fracture testing configuration and geometry of specimen

The universal testing machine (MTS 322) with closed-loop servo control was used to achieve a stable failure of specimens. The complete load-time curve was recorded to check the stability during the test. The mid-span deflection  $\delta$  of specimen and the applied load  $P$  were recorded continuously until the beam was completely separated into two halves. The test was performed with constant rate of deformation, so that the maximum load was reached within 30–60 seconds after start of the test. Fig. 3 shows the

testing machine with beam specimen.

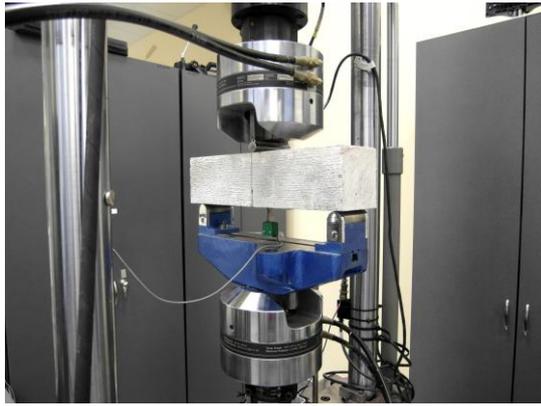


Fig. 3. Notched beam specimen during testing

The fracture energy ( $G_F$ ) is defined as the area under the load-deflection curve per unit fractured surface area. The fracture energy of fibre reinforced concrete tested in this experimental study, was evaluated using the equation given by RILEM TC 50-FMT [23], in which energy was calculated from load-deflection curves obtained by performing a three-point bending test

$$G_F = \left[ \int_0^{\delta_{max}} P(\delta) d\delta + mg \delta_{max} \right] / [(d - a_0)b], \quad (1)$$

where  $g = 9.81 \text{ m/s}^2$ ;  $d$  – beam depth;  $b$  – beam width;  $a_0$  – notch depth and  $\delta_{max}$  – maximum deflection. In connection with the test, the weight of the beam  $m$  was determined and included into calculation of  $G_F$ .

The experimental load-deflection data of three-point bending tests were used to obtain the softening function which characterizes the post-peak behaviour of concrete.

The flexural strength and elastic modulus were calculated from load-displacement relationships [21]. The flexural strength was defined by the load capacity at the first crack. Compressive strength of concrete was determined according to EN 12390-3 [24].

### 3. RESULTS

#### 3.1. Mechanical properties of composites tested

The mechanical properties of cement composite with synthetic fibers determined after 28 days of curing were presented in Table 3. The ranges of accuracy of test results were determined according to [25]. The material with synthetic fibres revealed higher modulus of elasticity in comparison to reference concrete without fibres. The modulus was determined on the basis of initial (ascending) part of load-deflection curve. Thus, the fibres had also an effect on pre-cracking behaviour of cement composite. However, the type and content of fibre, in the range considered, did not cause the significant differences in elastic modulus value. The incorporation of the fibres caused the slight increase in compressive strength, particularly in case of concrete with fibre types A and B. The average flexural strength of plain concrete was 4.0 MPa and incorporation of type A fibres appeared to increase it by about 18% for volume fraction 0.9 % (see Table 2). The increase in the flexural strength of concrete with shorter fibres type B were lower, by about 10 %.

Table 3. Mechanical properties of fibre reinforced concrete (average values and range of accuracy)

Fibre type	Fibre content, vol. %	Elastic modulus, GPa	Compressive strength, MPa	Flexural strength, MPa
non	0	18.4 ± 1.8	53.4 ± 3.8	4.0 ± 0.7
A straight 50 mm	0.3	23.2 ± 2.3	55.0 ± 3.0	4.4 ± 0.6
	0.6	23.8 ± 1.9	58.8 ± 4.2	4.6 ± 0.3
	0.9	24.6 ± 2.5	58.5 ± 6.1	4.8 ± 0.4
B straight 25 mm	0.3	22.3 ± 2.0	52.4 ± 4.8	4.0 ± 0.6
	0.6	23.8 ± 1.8	55.0 ± 5.4	4.3 ± 0.5
	0.9	24.3 ± 2.0	59.1 ± 4.0	4.4 ± 0.4
C fabric. 50 mm	0.3	23.6 ± 2.2	56.8 ± 6.2	4.3 ± 0.6
	0.6	22.6 ± 1.6	55.3 ± 3.5	4.2 ± 0.7
	0.9	23.7 ± 2.4	53.0 ± 4.3	4.3 ± 0.7

The fabricated fibres type C caused only the small change in flexural strength in comparison to a reference concrete. For dosage rates of fibers type C from 0.3 % to 0.9 % no significant differences in strength were observed. It may be concluded that incorporation of synthetic fibres did not significantly improve the flexural strength. The main benefit of using fibres was the improved ductility in the post-cracking region and it could be determined from load-deflection measurement.

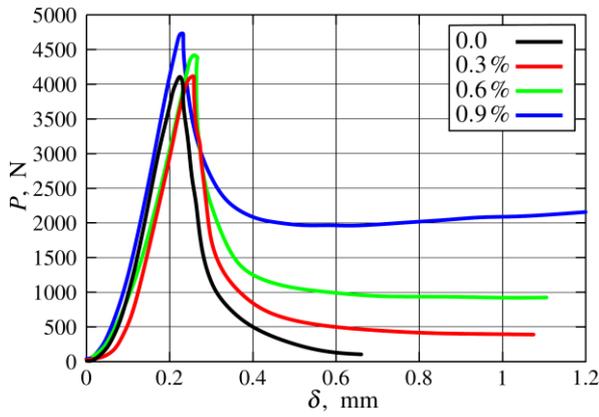
#### 3.2. Effect of synthetic fibre on fracture energy

The typical complete curves of the load  $P$  versus the displacement in the mid-span  $\delta$  for concrete with different types of fibres were presented in Fig. 4–Fig. 6, respectively. The analysis of  $P$ – $\delta$  plot for concrete with fibres makes it possible to investigate the changes in concrete properties related to the loss of brittle material character.

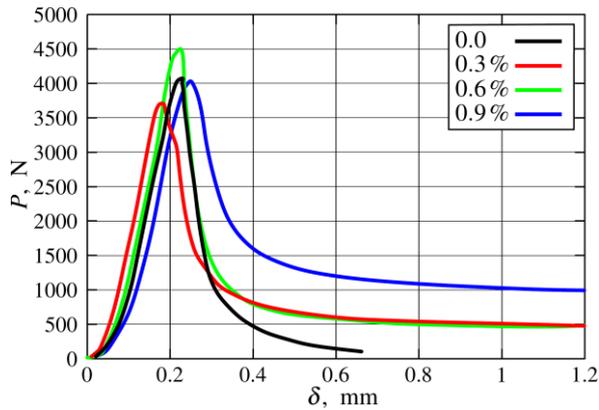
From the  $P$ – $\delta$  plot, it can be seen that the slope of initial part of the curve becomes more steep with the increase in the fibre content. This phenomenon has an effect on the Young modulus results. After the linear segment of  $P$ – $\delta$  curve, the load reaches the maximum value, which indicates the onset of crack initiation at the tip of the notch. The value of  $P_{max}$  recorded for composites with fibres A increased when the fibre volume fraction increased from 0 % to 0.9 %. For composites with various volume fraction of fibre types B and C the differences in  $P_{max}$  were not so significant and the value of maximum load depended on the strength of cement matrix and the incorporation of fibres was not very effective to counteract the crack initiation.

The increase in fibre content had no significant influence on the deflection  $\delta$  recorded for  $P_{max}$  (Fig. 4–Fig. 6). The deflections in the range from 0.185 to 0.260 mm were recorded for concrete specimens tested, independently of fibre dosage.

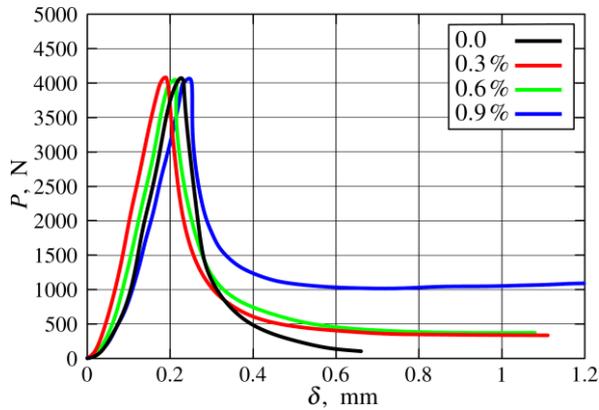
The descending part of  $P$ – $\delta$  plot was strongly influenced by both the type and content of fibres in concrete. For reference concrete the clear force drop after reaching the peak value and the softening behaviour were observed. For concrete with fibres, the general tendency was similar, but the fibres allowed transferring higher stresses at large crack openings. Thus, higher residual load carrying capacity at large deflection was recorded and a long plateau could be observed.



**Fig. 4.** Load  $P$  versus deflection  $\delta$  curves for concrete with various volume fraction of fibres type A



**Fig. 5.** Load  $P$  versus deflection  $\delta$  curves for concrete with various volume fraction of fibres type B



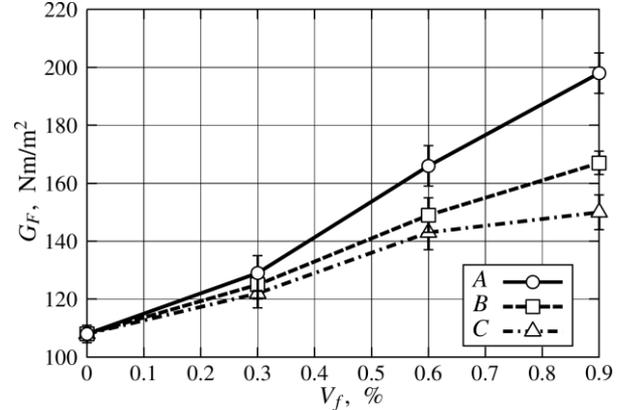
**Fig. 6.** Load  $P$  versus deflection  $\delta$  curves for concrete with various volume fraction of fibres type C

The load decreased much faster when dealing with low fibre-volume ratios. The residual load carrying capacity and the maximum deflection increased with the increase in fibre content.

The observed increase in fibre reinforced concrete ductility was the most significant in case of fibres type A with the length of 50 mm. The fibres of the same shape but shorter (type B with the length of 25 mm) and thicker fibres (structural type C with the length of 50 mm) were less effective. For fibre types B and C the relative improvement in residual load carrying capacity was reached for fibre dosage  $V_f=0.9\%$ . For higher fibre dosage a quasi-hardening behaviour was observed under bending. Considering the fracture energy is the product of load and deflection, the variation of  $P-\delta$  curve reflects the

variation of  $G_F$ .

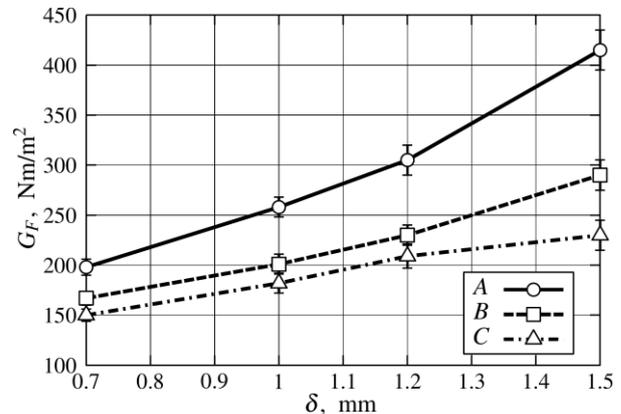
The changes in  $G_F$  with the increase in volume fraction of fibres tested are shown in Fig. 7. The specimens without fibres were destroyed for  $\delta=0.70$  mm. Thus, this value was assumed for calculation  $G_F$  aiming the comparison of fibre influence on fracture energy.



**Fig. 7.** Effect of fibre type and volume fraction  $V_f$  (%) on fracture energy  $G_F$  calculated for deflection  $\delta=0.70$  (vertical bars represent the range of accuracy)

The results presented in Fig. 7, in general, indicate that even in the range of  $\delta$  limited to 0.7 mm, growing fibre content has an increasing effect on the energy absorption. With the increase in fibre volume fraction,  $G_F$  is increasing gradually. However, the increase effect of fibre on  $G_F$  is not obvious when the fibre volume fraction is about 0.3 %, and the increase rate becomes greater for straight fibres (type A and B) than for crimped ones.

The changes in  $G_F$  for greater values of  $\delta$  were determined only for concrete with fibres. Fig. 8 shows the average dissipated energy  $G_F$  versus  $\delta$  for concretes containing 0.9 % of various fibres.



**Fig. 8.** Dissipated energy  $G_F$  versus  $\delta$  for concretes with different type of fibres ( $V_f=0.9\%$ ) (vertical bars represent the range of accuracy)

The dependency of dissipated energy  $G_F$  versus  $\delta$  for all concrete analysed was almost linear, regarding the scatter of measurements. The enhancement of energy absorption was strongly affected by the type of fibre reinforcement. The fibre type A appeared the most effective.

The reason why synthetic fibres can improve the fracture toughness of cement composite is related to specific characteristics of cement matrix and fibers. With lower elastic modulus and better flexibility, the bunched

polypropylene fibres can be distributed uniformly in cement matrix to form a three-dimensional disorderly supported space network structure when the fibers are mixed with the cement and fine aggregate. During the course of hardening, there will appear some micro-cracks because of the drying shrinkage and thermal shrinkage inside the cement matrix [1, 3].

When the micro-cracks are developing under the action of load, the polypropylene fibres stretching across the micro-cracks perform a bridging effect [6, 15]. The fibres can relieve the stress concentration on the top of the cracks and increase the resistance to crack development. Therefore, the fracture properties of cement composite are improved. The best results were obtained for flexible, ductile straight fibres with the length of 50 mm that improve the toughness and strain capacity in the post-cracking region. The thicker, rigid fibres were significantly less effective, although they had the specially prepared rough surface.

### 3.3. Post-cracking behaviour of composite

The data sets collected during test of specimens with different type of fibres were used to find the softening function describing the post-peak behaviour of synthetic fibre reinforced concrete with proper accuracy. Regarding the character of the softening curves obtained for concrete with fibres, the following expressions were employed to fit the descending part of  $P-\delta$  plot:

$$P(\delta) = \begin{cases} P_{\max} \left[ c_1 + (1-c_1) \left( \frac{\delta_p}{\delta} \right)^{c_2} \right] & \delta_p \leq \delta \leq \delta_s, \\ P_s + c_3(\delta - \delta_s), & \delta_s < \delta \leq \delta_k \end{cases} \quad (2)$$

where  $P_{\max}$  is peak load;  $\delta_p$  is deflection value corresponding to  $P_{\max}$ ;  $\delta_{\max}$  is maximum value of deflection,  $P_s$  and  $\delta_s$ —load and deflection value corresponding to turning point, where the curve becomes almost linear and the softening or quasi-hardening is observed. The coefficients  $c_1$ ,  $c_2$  and  $c_3$  are determined by means of the last-square method. The Eq. 2 and the way of fitting were presented in Fig. 9.

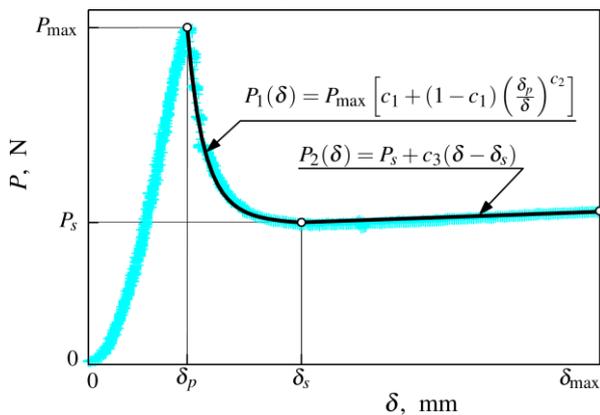


Fig. 9. Equations for experimental curve fitting

Two steps integration of Eq. 2 in the range from deflection value  $\delta_p$  corresponding to  $P_{\max}$  to  $\delta_s$  and then from  $\delta_s$  to the maximum value of deflection  $\delta_{\max}$  at final fracture and dividing the value obtained by unit fractured surface area, gives the value of  $G_F$  calculated on the basis

of post-cracking zone. The weight of the beam should also be included in the same manner as in Eq. 1.

The fracture energy was calculated according to Eq. 2 for all concrete tested, assuming  $\delta_f = 0.70$  and compared with the values obtained from the experiment. The value of  $\delta_f$  was limited to take into account the results obtained for concrete without fibres (marked 0 in Fig. 10). The result of the comparison, presented in Fig. 10, confirmed good fit of approximated function.

The shape of the initial part of softening curve, described by the power function in Eq. 2, is determined by  $c_2$  coefficient. The values of  $c_2$  increase with the increase in fibre dosage. The final part of softening curve, expressed by the linear function, is determined by  $c_3$  coefficient. The negative values of  $c_3$  are obtained for typical softening, while the positive values of  $c_3$  mean quasi-hardening behaviour of concrete with fibres.

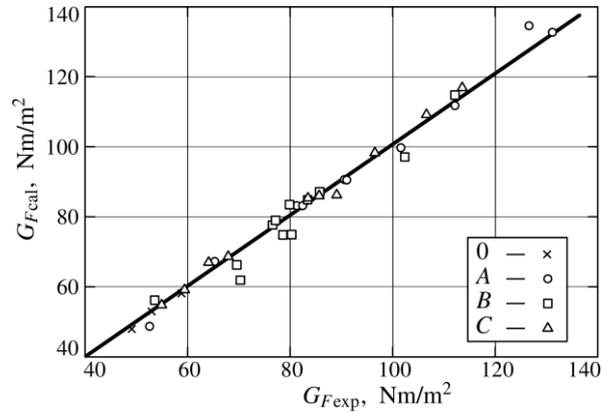


Fig. 10. Comparison between calculated  $G_{Fcal}$  and measured in experiment  $G_{Fexp}$  values

## 4. CONCLUSIONS

The results obtained from the test, performed on various synthetic fibre reinforced concrete, made it possible to analyse the fibre influence on fracture properties of concrete. The pre-peak behaviour of material was slightly affected by the addition of fibres to cement matrix. At this stage, the deformation regime was dominated by the cement matrix properties. The synthetic fibres caused a slight increase or an almost insignificant effect on the peak-load value as well as on the compressive and flexural strength of composite, but the post-peak behaviour revealed an important improvement compared to the reference concrete. Results of measuring toughness and energy-absorption characteristics showed that fibre reinforced specimens acquire a great ductile behaviour and energy absorption capacity, compared to ordinary concrete specimens. However, the beneficial effect of synthetic fibres depends on their length and flexibility. Stiff fibres (type C) were less effective than flexible ones (types A and B).

The experimental results were used as input data to perform the analysis which led to the simple function system describing the post-peak behaviour of concrete with fibres. The model gave results in very good agreement with experimental data in terms of load-deflection diagrams. On the basis of softening function obtained, the value of  $G_F$  can be calculated and analyzed. The function

obtained can be introduced in more complex numerical models to reproduce the behaviour of structural elements.

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### REFERENCES

1. ACI 544. IR-96. State-of-the-art report on fibre reinforced concrete. Manual of concrete practice. Farmington Hills, 1998.
2. **Meddah, M.S., Bencheikh, M.** Properties of Concrete Reinforced With Different Kinds of Industrial Waste Fibre Materials *Construction and Building Materials* 23 2009: pp. 3196–3205.
3. **Yoo, D.-Y., Park, J.-J., Kim, S.-W., Yoon, Y.-S.** Early Age Setting, Shrinkage and Tensile Characteristics of Ultra High Performance Fiber Reinforced Concrete *Construction and Building Materials* 41 2013: pp. 427–438.
4. **Bordelon, A.C.** Fracture Behavior of Concrete Materials for Rigid Pavements System. MA Thesis. Graduate College of University of Illinois at Urbana-Champaign, USA, 2007.
5. **Michels, J., Christen, R., Waldmann, D.** Experimental and Numerical Investigation on Postcracking Behavior of Steel Fiber Reinforced Concrete *Engineering Fracture Mechanics* 98 2013: pp. 326–349.
6. **Kalpokaite Dičkuvienė, R., Lukošiuūtė, I., Brinkienė, K., Baltušnikas, A., Čėsniėnė, J.** Applicability of the Waste Fibres in Cement Paste *Materials Science (Medžiagotyra)* 19 (3) 2013: pp. 331-336.
7. **Soutsos, M.N., Le, T.T., Lampropoulos, A.P.** Flexural Performance of Fibre Reinforced Concrete Made with Steel and Synthetic Fibres *Construction and Building Materials* 36 2012: pp. 704–710.
8. **Karihaloo, B.L.** Failure of concrete, in: Comprehensive structural integrity, Elsevier Pergamon, UK, 2003: pp. 477–548.  
<http://dx.doi.org/10.1016/b0-08-043749-4/02087-5>
9. **Shah, S.P., Swartz, S.E., Ouyang, Ch.** Fracture mechanics of concrete: Applications of fracture mechanics to concrete, rock and other quasi-brittle materials. John Wiley & Sons, Inc., New York, 1995: p. 552.
10. **Bazant, Z.P.** Concrete Fracture Models: Testing and Practice *Engineering Fracture Mechanics* 69 2002: pp. 165–205.  
[http://dx.doi.org/10.1016/S0013-7944\(01\)00084-4](http://dx.doi.org/10.1016/S0013-7944(01)00084-4)
11. **Zhao, Z., Kwon, S.-H., Shah, S.P.** Effect of Specimen Size on Fracture Energy and Softening Curve of Concrete: Part I. Experiments and Fracture Energy *Cement and Concrete Research* 38 2008: pp. 1049–1060.
12. **Kosior-Kazberuk, M.** Variations in Fracture Energy of Concrete Subjected To Cyclic Freezing and Thawing *Archives of Civil and Mechanical Engineering* 13 2013: pp. 254–259.
13. **Guo, X.-H., Tin-Loi, F., Li, H.** Determination of Quasibrittle Fracture Law for Cohesive Crack Models *Cement and Concrete Research* 29 1999: pp. 1055–1059.
14. **Buratti, N., Mazzotti, C., Savoia, M.** Post-cracking Behaviour of Steel And Macro-Synthetic Fibre-Reinforced Concretes *Construction and Building Materials* 25 2011: pp. 2713–2722.  
<http://dx.doi.org/10.1016/j.conbuildmat.2010.12.022>
15. **Voit, K., Kirnbauer, J.** Tensile Characteristics And Fracture Energy of Fiber Reinforced and Non-Reinforced Ultra High Performance Concrete (UHPC) *International Journal of Fracture* 188 2014: pp. 147–157.  
<http://dx.doi.org/10.1007/s10704-014-9951-7>
16. **Conforti, A., Minelli, F., Tinini, A., Plizzari, G.A.** Influence of Polypropylene Fibre Reinforcement and Width-To-Effective Depth Ratio in Wide-Shallow Beams *Engineering Structures* 88 2015: pp. 12–21.  
<http://dx.doi.org/10.1016/j.engstruct.2015.01.037>
17. **Behfarnia, K., Behravan, A.** Application of High Performance Polypropylene Fibers in Concrete Lining of Water Tunnels *Materials and Design* 55 2014: pp. 274–279.
18. **Zhang, P., Liu, Ch., Li, Q., Zhang, T.** Effect of Polypropylene Fiber on Fracture Properties of Cement Treated Crushed Rock *Composites: Part B* 55 2013: pp. 48–54.  
<http://dx.doi.org/10.1016/j.compositesb.2013.06.005>
19. **Wang, Z., Gao, D., Zhu, H., Zhang, Q.** Fracture Properties of Polypropylene Fiber Reinforced High Strength Concrete *Kuei Suan Jen Hsueh Pao (Journal of the Chinese Ceramic Society)* 35 (10) 2007: pp. 1347–1352.
20. **Zhang, P., Li, Q., Sun, Z.** Influence of Silica Fume and Polypropylene Fiber on Fracture Properties of Concrete Composite Containing Fly Ash *Journal of Reinforced Plastic Compositions* 30 (24) 2011: pp. 1977–1988.
21. Model Code 2010 Comité Euro-International du Béton *fib (CEB-FIP)* 2012.
22. RILEM TC 162-TDF. Test and Design Method for Steel Fibre Reinforced Concrete  $\sigma$ - $\epsilon$  Design Method *Materials and Structures* 36 2003: pp. 560–567.  
<http://dx.doi.org/10.1617/14007>
23. Recommendation TC 50-FMT RILEM. Determination of the Fracture Energy of Mortars and Concretes by Means of Three-Point Bend Tests on Notched Beams *Materials and Structures* 18 1985: pp. 285–290.
24. EN 12390-3. Testing Hardened Concrete: Compressive strength of test specimens, 2011.
25. **Brandt, S.** Data analysis. Statistical and computational methods for scientist and engineers. New York, Springer-Verlag, 1999: p. 692.