# Peculiarities of Packing Fraction of Open-packed Yarn Model 

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#### Abstract

The yarn packing properties of a hypothetical model of yarn open cross-sectional structure are discussed. The two different modifications of above-mentioned model are analysed. These modifications differ in number of filaments and magnitude of gaps left in ring layers. The indices of packing fraction of open-packed yarn are computed. The main peculiarities of these results are also described. The results of calculations confirmed that the packing fraction of openpacked yarn is almost constant for a wide range of yarn cross-sectional layers. Moreover, an assumption of constancy for magnitude of packing fraction was analysed for a case of twisted yarns. The hypothetical prediction of packing fraction was compared with the experimental results. Keywords: yarn, yarn structure, yarn cross-section, open packing, packing fraction.


## INTRODUCTION

The study of yarn geometry is a necessary part of textile materials science. From the engineering standpoint, the understanding of basic features of yarn geometry, especially of yarn packing, is a matter of great importance in problems of textile design. Usually the various models of yarn structure are served in that design [1-11]. The indices of yarn packing fraction are widely used in predicting of such yarn structural properties as overall density, diameter, twist contraction, linear density, etc. Moreover, the values of yarn packing are necessary in design of various textile products, namely complex yarn structures and fabrics. For example, the results of predicting of yarn properties from fibre properties are discussed up to now [12-14].

One of the basic forms of yarn packing models is the open packing [6]. Although real yarns differ from idealised structure of open-packed yarn model but in theoretical works such a model is more suitable in compare with empirical approach. A main reason of such an opinion lies in necessity of experimental data for making of empirical model. Various problems of open packing are discussed in papers $[1-4,6,11]$. Some results presented in these papers, for example, the indices of packing fraction differ in number. Unfortunately, the peculiarities and especially reasons of such results aren't explained in detail.

The aim of this study is to evaluate the packing properties of open-packed yarn model.

## RESULTS AND DISCUSSIONS

An analysis was made for a yarn, composed of equal, non-compressible and circular filaments (fibres). The filaments are arranged in ring-shaped layers (Fig. 1). A thickness of every ring layer is equal to the filament diameter. Therefore, for example, the filaments of third layer are only touching a circle, which restricts second layer surface. An analogous nature of arrangement of

[^0]filaments is character to the other layers. In some layers the gaps between the filaments are formed [6].

In order to define the properties of packing of filaments in a yarn, usually an index of packing fraction is used. According to various authors [3, 5-9] the packing fraction may be defined in some ways:

1. As the ratio of the volume of constituent filaments to the volume of the yarn;
2. As the ratio of the yarn density to the filament density;
3. As the ratio of total cross-sectional area of filaments in the yarn to cross-sectional area of the yarn.


Fig. 1. Cross-section of open-packed yarn model
In this study the packing fraction was counted up using above-mentioned third way. A suitability of this method is the best, as in this case it is possible to find all packing fraction results directly from scheme of crosssectional structure of yarn model. The packing fraction values for each ring layer $\Phi_{i}$ and for a whole yarn crosssection $\Phi$ were calculated using formulas presented in paper [10]:
$\Phi_{i}=A_{f i} / A_{y i}$
and
$\Phi=\left(A_{f 1}+\ldots+A_{f i}+\ldots+A_{f t}\right) /\left(A_{y 1}+\ldots+A_{y i}+\ldots+A_{y t}\right)=$
$=\sum A_{f i} / \sum A_{y i}$,
where $A_{f i}$ - total sum of cross-sectional area of all filaments in the current ring layer, $A_{y i}$ - cross-sectional area of the current ring layer, $i$ - current ring layer, $t$ number of the ring layers in the yarn.

The two different modifications of open-packed yarn model, named as Case I and Case II, are analysed in subsequent work. These modifications differ in number of filaments and magnitude of the gaps left in ring layers.

In the Case I a number of filaments in current layer $i$ was calculated as:
$n_{1}=K(i-1)$,
where $K$ - coefficient of proportionality.
The conditions for indices $K$ and $i$ are the following: $K=6$ and $i \geq 2$. A width of the gap between filaments in a ring layer may be greater in number or less in compare with the filament diameter. For example, for $i=5$ and $i=6$ the gaps equal $2.12 r_{f}$ and $2.84 r_{f}$ accordingly, where $r_{f}-$ filament radius. Therefore these gaps exceed the filament diameter. The gaps for third and forth layers $(i=3, i=4)$ of yarn cross-section are shown in above-mentioned Fig. 1. These gaps are less in compare with the filament diameter.

For the Case II a number of filaments in the current ring layer is of highest value but any deformation of these filaments takes place in transversal direction. Such modification was mentioned in [6]. For example, there are more filaments in fifth and sixth layers ( $i=5, i=6$ ) in compare with the Case I. Therefore a cross-sectional structure of yarn model in the Case II may be described as open with respect to different layers but as much as possible packed in the every layer.

The counting results of cross-sectional area of each ring layer and the data of cross-sectional area of yarn are presented accordingly in Fig. 2 and 3. It is not hard to notice the linear influence of number of the current ring layer $i$ (except $i=1$ for central filament) on the crosssectional area $A_{y i}$. The relation between index $t$ and $\sum A_{y i}$ is more complex as the linear relation exists between index $t$ and yarn diameter only.

The next two graphs (Fig. 4 and 5) show crosssectional area of filaments in current layer (index $A_{f f}$ ) and also in yarn (value $\sum A_{f j}$ ). The first difference between the above-mentioned Cases I and II arises at fifth and sixth layers.

The counting results of packing fraction for different ring layers are presented in Table 1. These results reveal that in the Case I the packing fraction $\Phi_{i}$ is constant if $i \geq 2$. In the Case II the quantity of index $\Phi_{i}$ is not stable from the fifth layer. So in the Case I is still more reasonable to assume that packing density is uniform across the yarn section. Another peculiarity is that the packing fraction $\Phi_{i}$ for the Case I is less than for Case II (except $i=1, \ldots, 4$ ). This property also relates with cause of different arrangement of filaments in the ring layers.

Table 2 summarises the results of packing fraction $\Phi$ of open-packed yarn, where $t$ - number of the ring layers in the yarn.


Number of current ring layer $i$

Fig. 2. Calculated values of cross-sectional area for different ring layers of open-packed yarn


Fig. 3. Calculated values of cross-sectional area of open-packed yarn
In the Case I the packing fraction $\Phi$ is not constant only for a small number of filaments $n$ or for corresponding number of layers $t$. With the further increasing of $t$ or $n$ the packing fraction $\Phi$ is almost constant. For example, if index $t$ increases from 5 to 12 (or index n increases from 61 to 397), the index $\Phi$ falls only $0.4 \%$. So the same value of yarn packing fraction ( $\Phi=0.75$ ) may be used not only in investigations of ordinary multifiliament or spun yarns but in studies of microfilament yarns too.

In the Case II, when all the layers are packed at most, the packing fraction $\Phi$ for $t \geq 5$ (Table 2) slightly exceeds the previous results. For example, for $t=5$ a difference is $1.6 \%$, for $t=6$ this value is $2.3 \%$.


Fig. 4. Calculated values of cross-sectional area of filaments for different ring layers


Fig. 5. Calculated values of cross-sectional area of filaments in open-packed yarn
Table 1. The results of packing fraction for different ring layers of open-packed yarn

| $i$ | $\Phi_{i}$ |  |
| :---: | :---: | :---: |
|  | Case I | Case II |
| 1 | 1.000 | 1.000 |
| 2 | 0.750 | 0.750 |
| 3 | 0.750 | 0.750 |
| 4 | 0.750 | 0.750 |
| 5 | 0.750 | 0.781 |
| 6 | 0.750 | 0.775 |

In addition the packing fraction results according to [1] are presented in the Table 2. The authors of this paper did not indicated the formulas they used but these results are in very good agreement with the calculated values in accordance with the Case I.

Table 2. The results of packing fraction of open-packed yarn

| $t$ | $\Phi$ |  |  |
| :---: | :---: | :---: | :---: |
|  | Case I | Case II | According to [1] |
| 1 | 1.000 | 1.000 | - |
| 2 | 0.778 | 0.778 | 0.778 |
| 3 | 0.760 | 0.760 | 0.760 |
| 4 | 0.755 | 0.755 | 0.755 |
| 5 | 0.753 | 0.765 | 0.753 |
| 6 | 0.752 | 0.769 | 0.752 |
| 7 | 0.751 | - | 0.753 |
| 8 | 0.751 | - | 0.751 |
| 9 | 0.751 | - | 0.751 |
| 10 | 0.751 | - | - |
| 11 | 0.751 | - | - |
| 12 | 0.750 | - | - |

It is worth to notice here one more circumstance. As a rule, an assumption of constancy for magnitude of packing fraction $\Phi$ is used in various theoretical investigations of twisted yarns. For instance, Binkevičius [3] made this assumption in prediction of twist contraction of yarns. Unfortunately, a phenomenon of such assumption wasn't explained in detail.

We solved this problem using the following way. If a twist is imparted to a yarn, an angle between axial line of each filament and yarn axis isn't equal to zero. Therefore a crossing line of each filament in cross-section of single twisted yarn has an elliptical shape. If we assume that the filaments are inextensible during twisting, a thickness of each ring layer is equal to filament diameter $d_{f}$, that's to say to a semi-minor axis of the ellipse. A length of semimajor axis $d_{f}$ consists of its degree of slope. An equation for deriving the semi-major axis, using 'mean' cosine of inclination of filaments is
$d_{f}{ }^{\prime}=d_{f} / \overline{\cos } \beta \mathrm{o}$,
where $\beta$ o - single yarn surface helix angle.
Therefore a number of filaments, which lie into the same ring layer, is less now. For example, if $d_{f}{ }^{\prime} \approx 1.5 d_{\mathrm{f}}$, there are approximately 8 filaments instead 12 filaments in the third layer. Consequently the number of layers in twisted yarn model is increased and, moreover, a diameter of this yarn is increased. It is no hard to understand that in such conditions a packing fraction $\Phi$ remains in fixed level if
$\sum A_{f j}=(1 / \overline{\cos } \beta \mathrm{o}) \sum A_{f i}$
and
$\sum A_{y j}=(1 / \overline{\cos } \beta \mathrm{o}) \sum A_{y i}$,
where $A_{f j}, A_{f i}$ - total sum of cross-sectional area of all filaments in current layer $j$ (for twisted yarn) and in current layer $i$ (for zero twist yarn); $A_{y j}, A_{y i}$ - cross-sectional area of current layer $j$ (for twisted yarn) and current layer $i$ (for zero twist yarn).

As it was mentioned earlier, real yarn structures have deviations from ideal forms. Various reasons of different packing were explained in detail in paper [5]. The packing fraction, which was obtained in experimental way, changes
in wider range. For example, the authors of paper [1] refer to the packing fraction from 0.23 for zero twist yarns to 0.70 for highly twisted yarns. According to Neckar and Jezek [11] index $\Phi$ fluctuates for highly twisted multifilament yarns from 0.75 to 0.85 and for spun yarns from 0.35 to 0.55 . Still wider range of packing fraction ( $0.3-0.9$ ) is presented in paper [6]. The results of ratio of yarn density to fibre density, which is none other than packing fraction, for filament yarns in paper [9] (0.5-0.8) are also similar to previously mentioned. However, we can notice in [6] that in the wide range of twist the value of $\Phi$ is rather stable and comparatively not much differs from theoretical value. Therefore rather often the packing fraction obtained for hypothetical open-packed model may be applied as a constant for twisted yarns with good reason.

## CONCLUSIONS

The two different modifications of open-packed yarn model were considered. The one modification, named as Case I, may be described not only as open-packed, but as with gaps (a width of a gap may to exceed filament diameter) between filaments in some ring layers. Another modification, named as Case II, may be considered as open with respect to different layers but as much as possible packed in every layer. Therefore the gaps in the Case II are less in compare with the Case I.

Counting results confirmed that the packing fraction of each type of open-packed yarn is almost constant for wide range of yarn cross-sectional layers. Therefore the same fixed value of yarn packing fraction may be used for microfilament yarns, as it is used for ordinary multifilament or spun yarns.

The study showed that there are reasons to assume the value of the $\Phi=0.75$ for open-packed yarn model modification, named as Case I. In the Case II the packing fraction slightly exceeds the packing fraction results obtained for the Case I.

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