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Surface Roughness Simulation Using Fractal Interpolation to the Profilogram

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This paper presents the methodology and results of surface roughness simulation of soft polymer materials using approximations of fractal interpolation curves. The modeling algorithm and its realization are made so that they can be applied to any profilogram. The comparison of created models and real profilograms is done using autocorrelation functions and fractal dimensions. The three-dimensional model of the roughened surface is developed and equations, relating the area of the surface to the characteristics of abrasive materials, i.e. to the size of an abrasive grain, are obtained. Those equations are to be used in forecasting the area of the surface of a butadiene-styrene rubber evading the experiment itself.

Keywords: abrasion, surface roughness, three-dimensional model of a rough surface, autocorrelation function, fractal interpolation, fractal dimension.

1. INTRODUCTION

The real contact surface area an adhesive joint plays an important role in its strength and durability. This parameter depends not only on the geometrical dimensions of the surface glued, but also on the roughness of the surface [1 – 3]. It is almost impossible to find the real area of a rough surface by applying direct methods. Such area could be approximately evaluated from the numerical model of the surface. In order to forecast the relation between the real area of a rough surface and the regimes of surface abrasion it is necessary to find the relationship between the real contact area and the indicators of surface roughness. The size of an abrasive grain of the abrasive material, which is used for surface roughening, has influence on the characteristics of the surface roughness. These characteristics may be of two types: statistical and fractal, depending on positions the profile of the roughened surface is treated. Statistical characteristics of the profile are obtained by treating the profile of the surface roughness as the realization of a stochastic process [4]. The assumptions of a stationary, ergodic and normally distributed process are also considered. However, those assumptions are not always satisfied and researchers are encouraged to develop other models of surface profile [5, 6].

Currently, methods of fractal geometry and important quantitative property of fractals, fractal dimension, are used to define surface roughness [7, 8]. It is important to create a surface roughness model depending on a specific parameter characteristic for that surface only. Such a parameter could be the fractal dimension of the profile of the rough surface [9 – 11]. The fractal dimension indicates how densely an object appears to fill the space. The apparatus of fractal interpolation enables us to obtain a rough surface model with different grades of roughness, since particular fractal interpolation parameters have influence on the fractal dimension of the surface profile.

The three-dimensional model of surface roughness described in this paper employs a quality of self similarity of a two dimensional curve. The real profilogram may not hold this quality, but the model is formed in a way that it should correspond to the real profilogram in its shape and fractal dimension. Consequently, there are no extra conditions for profilogram to satisfy (in order to be modeled).

2. FRACTAL INTERPOLATION CURVES AND THEIR ESTIMATES

In this paper, the presented model performs first two iterations of fractal interpolation for the set of points $\{(x_i, y_i) \mid i = \overline{1, N}\}$, chosen from a profilogram. A simple way to perform fractal interpolation is to apply shear transformations [12].

The number of (two dimensional and bounded) shear transformations equals the number of intervals between the interpolation points. The i-th ($i = \overline{1; N-1}$) shear transformation is written as

$$\omega_i(x) = \begin{pmatrix} a_i & 0 \\ c_i & d_i \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix}, \quad (1)$$

here a_i, c_i, d_i, e_i and f_i are parameters of the transformation.

The latter parameters are found from the system bellow:

$$\begin{cases} \begin{pmatrix} a_i & 0 \\ c_i & d_i \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix} = \begin{pmatrix} x_{i-1} \\ y_{i-1} \end{pmatrix}; \\ \begin{pmatrix} a_i & 0 \\ c_i & d_i \end{pmatrix} \cdot \begin{pmatrix} x_n \\ y_n \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix}. \end{cases} \quad (2)$$

It follows that

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$$\begin{cases} a_i = \dfrac{x_i - x_{i-1}}{x_n - x_0}; \\ c_i = \dfrac{y_i - y_{i-1}}{x_n - x_0} - d_i \dfrac{y_n - y_0}{x_n - x_0}; \\ e_i = \dfrac{x_n x_{i-1} - x_0 x_i}{x_n - x_0}; \\ f_i = \dfrac{x_n y_{i-1} - x_0 y_i}{x_n - x_0} - d_i \dfrac{x_n y_0 - x_0 y_n}{x_n - x_0} \end{cases} . \quad (3)$$

Suppose, B is a continuous curve (initial set) connecting two interpolation points, (x_1, y_1) and (x_N, y_N).

Parameters a_i and d_i control the level of stretch of the initial set along the abscissa and the ordinate axes, respectively. The parameters d_i are arbitrary, although $|d_i| < 1$ must hold. The values c_i adjust the level of shear of the initial set in a particular interval. The parameters e_i and f_i correspond to the displacement of the initial set.

The first step in performing fractal interpolation is to find the union of transformations ω_i acting upon the initial set B:

$$W(B) = W^{01}(B) = \omega_1(B) \cup \omega_2(B) \cup \ldots \cup \omega_{N-1}(B). \quad (4)$$

The resulting set of points $W^{01}(B)$ is treated as the initial set and is transformed using the same transformation W, i. e. $W^{02}(B) = W(W(B))$.

The fractal interpolation function (FIF), corresponding to the set B, is defined to be:

$$FIF = \lim_{n\to\infty} W^{0n}(B), \; W^{0n}(B) = W\left(W^{0(n-1)}(B)\right). \quad (5)$$

An illustrative example is presented in Fig. 1. Here, the set of five interpolation points, $(0;0)$, $(3;2)$, $(6;1.5)$, $(10;2.5)$, $(12;1)$, is considered.

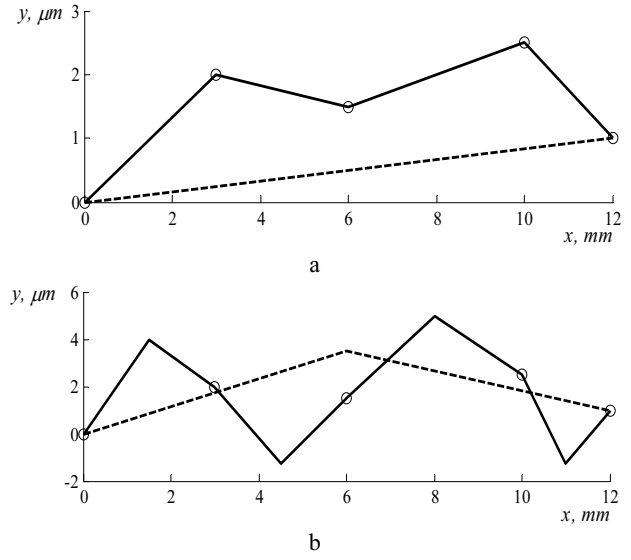


Fig. 1. First iteration of fractal interpolation of the data: a – the initial set B is a segment, b – the initial set B is a polygonal line

Fig. 2 shows the resulting curve after applying 2 iterations to the data depicted in Fig. 1.

It should be noted that FIF does not depend on the initial set B. If we restrict ourselves with two (three, or more) iterations, the fractal interpolation function (FIF) becomes dependent on the initial set B (Fig. 2).

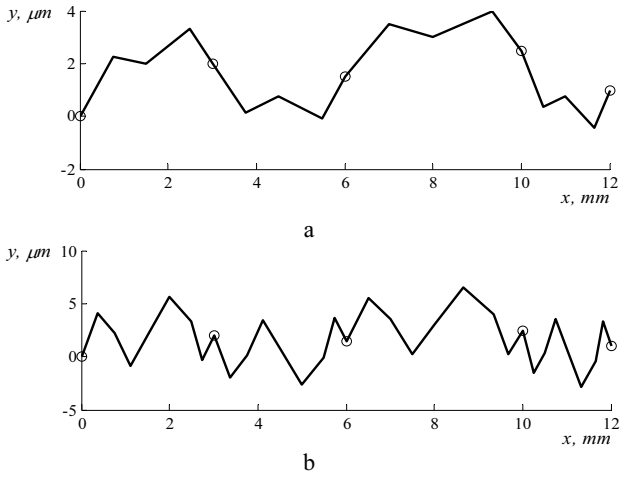


Fig. 2. Two iterations of the fractal interpolation: a – the initial set B is a segment, b – the initial set B is a polygonal line

3. RESULTS AND DISCUSSION

3.1. The experiment

The objects of the experiment are the monolithic rubber samples (butadiene-styrene rubber with the density and hardness according Shore A, respectively $\rho = 1.25$ g/cm^3 and H = 75 a.u.), whose surface was abraded with abrasive paper of different grades. The pressure force of 80 N has been used in the process of abrasion.

According to the FEPA's (Federation of European Producers of Abrasives) standard standing in the European Union, abrasive materials, with respect to their graininess, are numbered as follows: *P*24, *P*60, *P*1000, etc. The greater the grade of the abrasive paper, the smaller the size of the abrasive grain.

In this work, the following grades of abrasive paper are used: *P*24, *P*40, *P*60 and *P*100. The average diameter of an abrasive grain for each grade of the abrasive paper is shown in Table 1.

Table 1. The relation between the grade of an abrasive paper and the average diameter of an abrasive grain

The grade of the abrasive paper according to the FEPA	The average diameter of an abrasive grain, (mm)
*P*24	0.698
*P*40	0.382
*P*60	0.260
*P*100	0.149

In this paper, the roughness is treated as a whole of micro roughness located relatively close to each other. From this viewpoint, a surface processed with the grade P100 abrasive paper is rougher than the one processed with the grade P24 abrasive paper.

All profilograms were tested by Hommelwerke T500 surface finish tester – profilograph (Germany; minimal limit of measuring is 0.2 μm) perpendicular to the direction of abrasion. The profilograms of the surfaces abraded with the different grade of the abrasive paper are shown in Fig. 3.

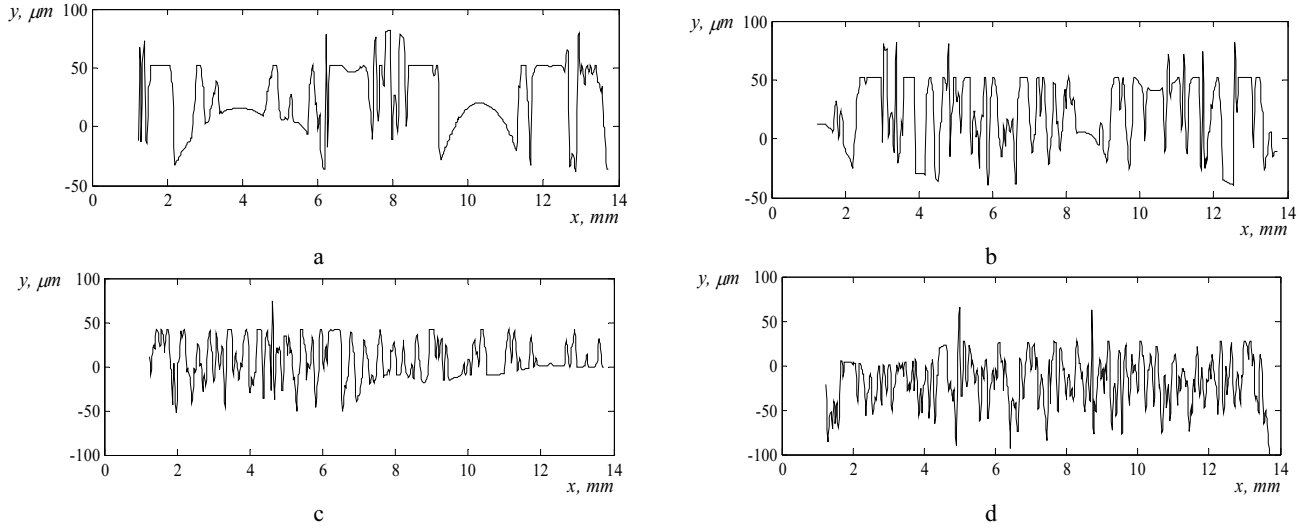


Fig. 3. The typical profilograms of a rough surface corresponding to the different grade of roughness: a – *P*24; b – *P*40; c – *P*60; d – *P*100

3.2. The simulation of a profilogram

The profile of surface roughness in the sample interval of length $l = 10$ mm is expressed as a set of points M_i, $i = \overline{1; N_0}$, with coordinates (x_i, y_i) defined.

To simulate a particular profilogram, at first a certain amount of its points must be chosen. These can be specific points (peaks or minimums) with respect to the ability to preserve the shape of the profilogram. But again, this depends on a particular profilogram and its shape.

There are lots of specific points in an extremely rough profilogram. They often reside beside each other in some areas of the profilogram and they can be far from each other in other areas. In such a case, taking of the union of shear transformations leads a huge number of new points in areas where the profilogram is extremely rough, i. e. the roughness of the model would be inadequately uneven. Because of this property, it is not advisable to choose specific points from the profilogram to perform fractal interpolation. When performing the research, the chosen points were distributed uniformly along the 0*x* axis, at the distance *u* from each other.

Bellow, we demonstrate the methodic of simulation of a profilogram by considering the excerpt from the profilogram, which represents the surface abraded with the grade *P*24 abrasive paper. The real profilogram and the selected points for simulation are depicted in Fig. 4.

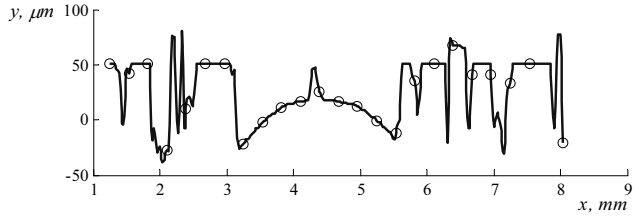


Fig. 4. The profilogram of surface roughened using abrasive paper *P*24 and the set of selected points for the simulation

It is important to decide upon the number of points (from the profilogram) to perform fractal interpolation. Often, we are forced to predict the resulting number of points in the model. Otherwise, the calculation time could be wasted or model may have too many or too few details compared to the data.

By performing fractal interpolation and using various initial sets (a segment, a polygonal line), it was found out that a polygonal line comprising 2 segments fits best for simulation of the profilogram. Such a selection of an initial set is useful for ensuring the model's adequacy to the data.

The total number of resulting points in the model is found by applying simple calculations. For example, if there are N interpolation points and a polygonal line is chosen as the initial set, the total number of points after the k-th iteration equals $f(N,k) = 2(N-1)^k + 1$. The simulation generates the number of resulting points, which is somewhat equal to the number of points N_0 in the profilogram ($f(N,k) \approx N_0$). Thus, for 2 iterations we have:

$$N = 1 + \sqrt{0.5(N_0 - 1)}. \quad (4)$$

According to the analysis completed, it was noticed that there is no need to perform many iterations, as it results in a huge number of resulting points in the model. If even 3 iterations are performed, the model gets extremely rough. On the other hand, the reduction of the number of interpolation points leads to the loss of the information of the profilogram's shape, and does not solve the issue.

The shear transformations, used to perform the first iteration of fractal interpolation, are depicted in Fig. 5.

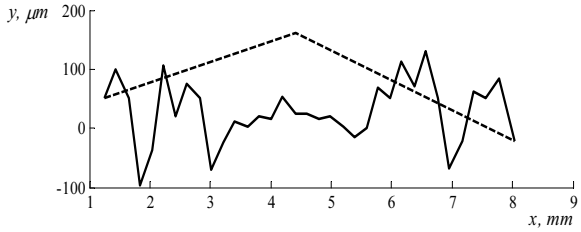


Fig. 5. The result of the first iteration applied to the chosen points from the profilogram

Parameters d_i of shear transformations must be selected in a way that the model roughness were similar to that of the profilogram. The rougher the profilogram is, the

greater d_i should be chosen in the case of smooth profilograms, values of d_i should be close to 0.

Roughness of the surface could be measured by calculating the standard deviation of the profile's ordinates y_{i_k}, $k=\overline{1;n}$, $n=N_0/N$. Thus, parameters d_i are calculated this way:

$$d_i=\sigma_i\left(y_{i_1},y_{i_2},...,y_{i_n}\right)/c; \quad (5)$$

here: i is the index of the interval between the chosen points for interpolation; n is the number of the profilogram's points in this interval and c is a constant. In other words, parameters d_i are ratios of the standard deviation σ_i of the ordinates y_{i_k} in a particular interval and the arbitrary constant. By performing experiments with various values of c and evaluating the model similarity to the profilogram, the value $|c|=50$ was chosen. The constant c is positive in a particular interval if the mean of profilogram's ordinates on this interval is higher than the arithmetical mean of the first and the last ordinates of the same interval and vice-versa. So, $\mu_i\left(y_{i_1},y_{i_2},...,y_{i_n}\right)>\mu_i\left(y_{i_1},y_{i_n}\right)$ makes $d_i>0$.

After a finite number of iterations the resulting curve is treated as the model of the profilogram. Two iterations transform the initial set to the curve depicted in Fig. 6.

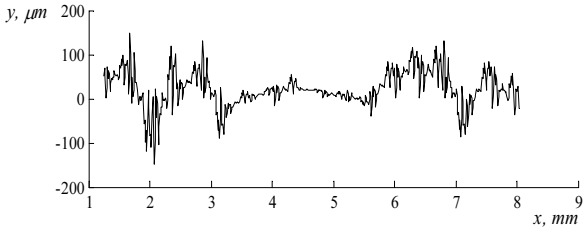


Fig. 6. Simulated profile of the roughened surface (using abrasive paper *P*24) after 2 iterations

The aforesaid technique of profilogram simulation and other calculations have been programmed using MATLAB 7.9.0 software.

In order to evaluate the model adequacy, the autocorrelation functions of the profilogram and its model have been used [13]. The autocorrelation functions of the profilogram and its model are calculated according to the formula:

$$ACF(\tau)=E\left(\left(y_t-\mu\right)\left(y_{t+\tau}-\mu\right)\right)/\sigma^2, \quad (6)$$

where τ is an increment, y_t, $t=\overline{1;N_0-\tau}$ – ordinates of the profilogram and μ, σ^2 are the mean and the variance of the ordinates y_t. The value of the autocorrelation function is dimensionless.

By comparing visually, the autocorrelation functions of the model and the profilogram, it is observed (Fig. 7) that they are very similar, provided τ is less than about 150. Thus, the original profilogram and its model correspond to each other. It was obtained that both autocorrelation functions could be approximated by the same analytical expression:

$$ACF(\tau)=\left(1+(\alpha\cdot\tau)^2\right)^{-1}, \quad (7)$$

where α is a parameter, obtained by applying the least squares method.

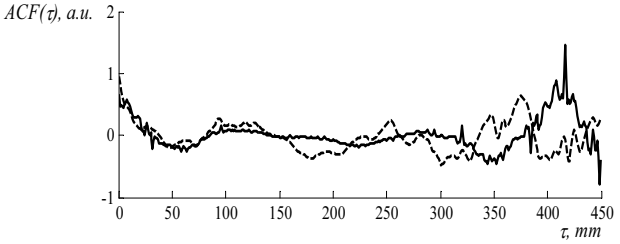


Fig. 7. The comparison of the autocorrelation functions: ---- – *ACF* of the real profilogram, —— – *ACF* of the simulated profilogram

Table 2 shows that the relative error (*RE*) between the autocorrelation functions of the original and modelled profilograms increases when the grade of the abrasive paper increases, provided the initial set for fractal interpolation is a segment. In the case of a polygonal line, the shape of the profilogram is preserved better if parameters d_i are calculated in the way of being proportional to standard deviations of the profilogram's ordinates on each interval.

3.3. The model of the three-dimensional surface

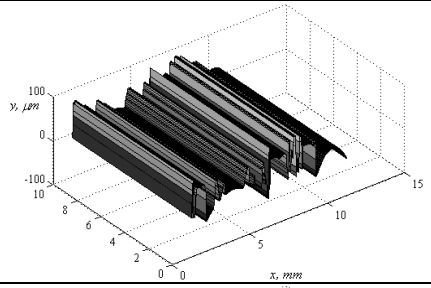
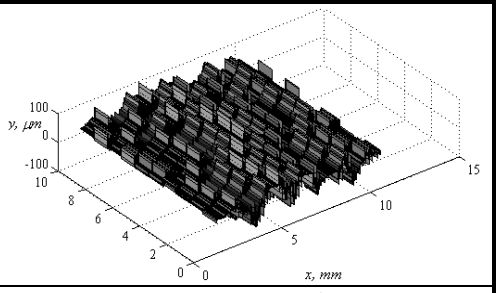
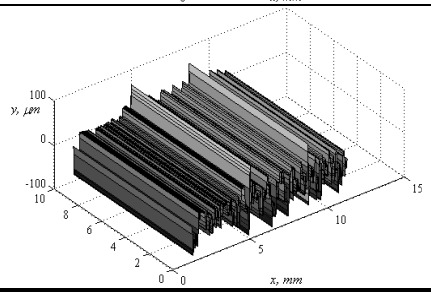
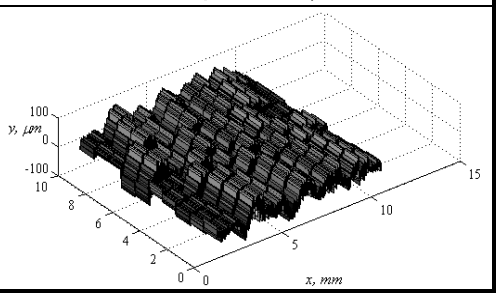
Let us designate the array of ordinates of the surface as $\{p_1, p_2, \text{K}, p_{N_0}\}$. Let N_0 represent the number of points comprising the profilogram. Due to the fact that the profilogram is a cut of a particular three-dimensional (3D) surface, setting the width enables us to make the 3D interpretation of surface roughness (Model 1).

While making the three-dimensional model of a profilogram, we also set the width to the estimate of its FIF. If $\{f_1, f_2, \text{K}, f_N\}$ are the ordinates of the FIF's estimate of the profilogram, then the three-dimensional interpretation of surface roughness is written in a matrix

Table 2. The mean squared (*MSE*) and the relative (*RE*) errors between the autocorrelation functions of the real profilogram and its model

The grade of the abrasive paper according to the FEPA	A segment is the initial set for interpolation		A polygonal line consisting of 2 segments is the initial set for interpolation	
	MSE	*RE*	*MSE*	*RE*
*P*24	0.134	8.068	0.339	20.382
*P*40	0.199	17.391	0.210	18.338
*P*60	0.211	19.112	0.298	26.986
*P*100	0.195	16.562	0.181	15.321

Table 3. The simulation of profilograms having a different grade of roughness.

The grade of the abrasive paper according to the FEPA	"Model 1"	"Model 2"
*P*24		
*P*100		

form as follows

$$F_{3D} = \begin{bmatrix} f_{11} & f_{12} & \text{K} & f_{1N_0} \\ \text{M} & \text{M} & \text{O} & \text{M} \\ f_{N_1 1} & f_{N_1 2} & \text{K} & f_{N_1 N_0} \end{bmatrix}; \quad (8)$$

here N_1 is arbitrary and denotes the number of zones in the model ($N_1 > 2$) and $f_{ij} = f_j$, $\forall i = \overline{1, N_1}$.

The surface is divided into zones along the direction of the profilogram, when the model of surface roughness (Model 2) is analyzed. We set a random width for each of the zones. Then each zone is randomly moved along the positive direction of the 0*x* axis by the value which is uniformly distributed on the interval $(0.1;0.3)$. The result of this process is depicted in Fig. 8.

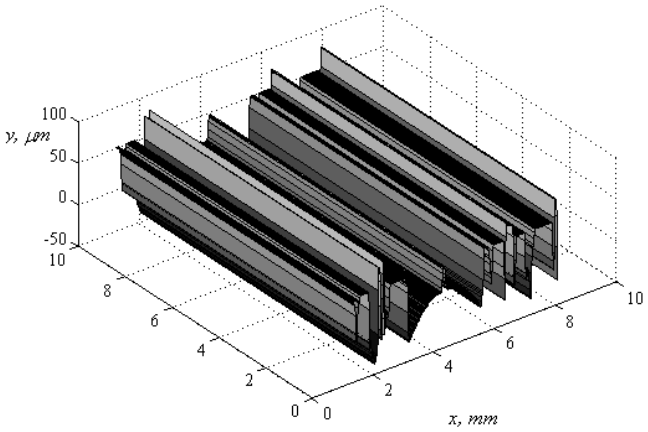
Finally, the extra parts of the surface zones are displaced to the front of the model where a free space resides now.

The three dimensional model of surface roughness is fractal-stochastic. It is due to the randomness of its coordinates and the self-similarity of the model's profile. Creation of it involves the random selection of the points of the profilogram; nevertheless, the zones in which we divide the model are randomly offsetted in space.

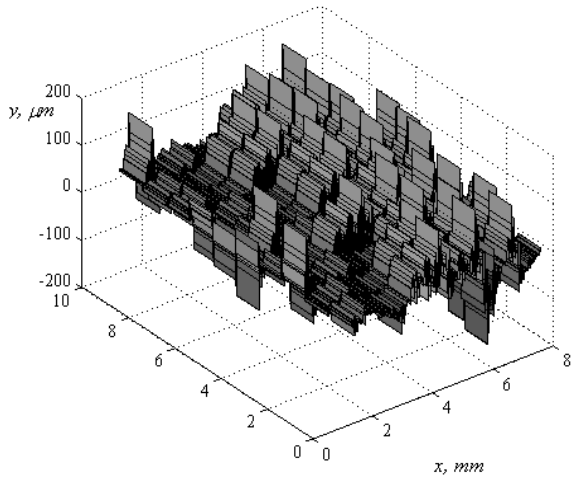
3.4. The comparison of models with different surface roughness

Each type of the profilogram has been investigated in order to compare the models of surface roughness. Table 3 shows models formed by performing two iterations of fractal interpolation. A polygonal line comprising of 2 segments has been chosen as the initial set for fractal interpolation. The surface model covers the real area of 100 mm^2 on the plane *x*0*z*.

Fractal dimension has been used to compare the model of a profilogram to the real profilogram. Such a comparison is given in Table 4. A segment as the initial set



a



b

Fig. 8. The 3D models of surfaces: a – Model 1, b – Model 2

of fractal interpolation has been used, 2 iterations have been performed and the area of the surface modelled is 10 mm × 10 mm.

Table 4. Comparison of models of the surfaces

The grade of the abrasive paper according to the FEPA	The surface area of „Model 1“ S_p, mm^2	The surface area of „Model 2“ S_m, mm^2	The estimate of fractal dimension of the real profilogram, D_p	The estimate of fractal dimension of a model of the profilogram, D_m
*P*24	101.014	101.359	1.713	1.685
*P*40	101.552	101.838	1.739	1.612
*P*60	103.302	101.754	1.800	1.644
*P*100	105.074	103.369	1.762	1.681

The three-dimensional models considered cover the same area (100 mm^2) on the plane $x0z$. By multiplying the length of the profilogram's model by its width, the surface area is calculated. The areas of surfaces "Model 1" and "Model 2" are denoted by S_p and S_m, respectively. The area S_p is greater for the surface modeled according to the rougher profilogram. Table 4 shows that using a segment (as the initial set) leads to the estimates of fractal dimensions D_p and D_m being comparable between the profilograms of grade 24 and 100 and their models.

Table 5 shows the comparison of the models of surface roughness when a polygonal line comprising 2 segments is chosen. Using the polygonal line (as the initial set) results in higher precision of the models (D_p and D_m differ less). The profilogram of grade 24 is an exception. This occurs due to the use of the polygonal line which makes the model of the profilogram rougher.

3.5. The relation between the area of the rough surface and the grade of the abrasive paper

The strength of the adhesive joints depends on the area of contact surfaces. This indicator depends not only on the geometrical dimensions of the substrate, but also on the surface roughness. If the appropriate receipt is considered, the glue can fill up all the roughness originated in the process of surface grinding or polishing. The resulting roughness of the surface depends on its inner structure, as well as on the roughness of the abrasive paper. Then the assumption about the relation between the grade of the abrasive paper and the surface could be drawn.

We have analyzed the relation between the area of the rough surface and the grade of the abrasive paper *r,* referring to the assumption above. We showed that the greater the size of an abrasive grain (i. e. the smaller the grade of the abrasive paper), the smaller the area of the roughened surface (Fig. 9). The relationship between the real area of the contact S_p and the area of the original surface S_0, can be written as follows:

$$S_p = S_0\left(0.9981 + \frac{0.0115}{R}\right); \tag{9}$$

here: $S_0 = 100$ mm^2, $R = \frac{r}{r_s}$, r – the average diameter of an abrasive grain (see Table 1), r_s is the diameter of an abrasive grain corresponding to the smallest grade of the abrasive paper.

According to the calculated coefficient of the determination, this relationship predicts the data with an accuracy of 97.36 %.

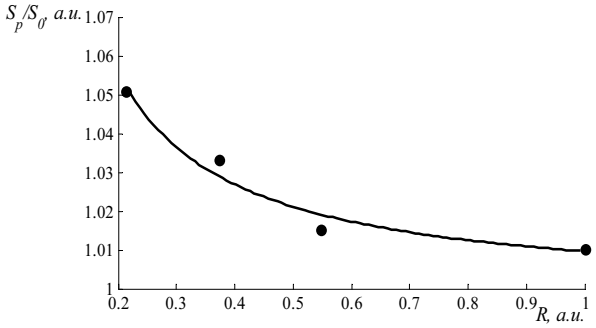


Fig. 9. The relationship between the area S_p and the value R

While forecasting the area S_m, the linear relation fits better:

$$S_m = S_0(1.086 - 0.052R). \tag{10}$$

According to the calculated coefficient of the determination, above relationship (Fig. 10) predicts the data with an accuracy of 98.46 %.

Due to the fact that the grade of the abrasive paper is always known, the equations obtained enables the researcher to predict the area of surface roughness, affected by the abrasive paper of any grade without performing the experiment itself.

Table 5. Comparison of models of the surfaces

The grade of the abrasive paper according to the FEPA	The surface area of „Model 1“ S_p, mm^2	The surface area of „Model 2“ S_m, mm^2	The estimate of fractal dimension of the real profilogram, D_p	The estimate of fractal dimension of a model of the profilogram, D_m
*P*24	101.014	103.463	1.713	1.672
*P*40	101.552	105.423	1.739	1.663
*P*60	103.302	106.743	1.800	1.680
*P*100	105.074	107.594	1.762	1.739

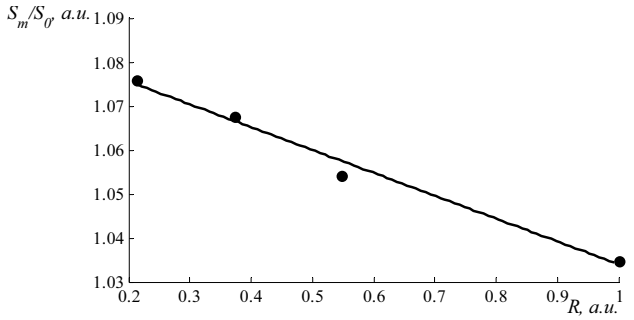


Fig. 10. The relation between the area S_m and value R

4. CONCLUSIONS

The use of fractal interpolation for modelling surface roughness has been investigated in the paper. It has been determined that using polygonal lines as initial sets for fractal interpolation is preferable while constructing the model of a profilogram.

The model obtained corresponds to the real profilogram in terms of both the shape of profile and the degree of filling the space. It was found by calculating the autocorrelation function and the estimate of fractal dimension for the real profilogram and its model.

The relationship between the real area of the rough surface of a butadiene-styrene rubber and the grade of the abrasive paper has been determined. So, the researcher is able to predict the area of surface roughness, affected by the abrasive paper of any grade, without performing the experiment itself, because the grade of the abrasive paper is always known.

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