ISSN 1392–1320 MATERIALS SCIENCE (MEDŽIAGOTYRA). Vol. 16, No. 2. 2010

Non-destructive Identification of Directions of Orthotrophy of Paper and Paperboard

Kazimieras RAGULSKIS [1], Edmundas KIBIRKŠTIS [2]*, Vaidas BIVAINIS [2], Asta KABELKAITĖ [2], Liutauras RAGULSKIS [3]

[1] *Lithuanian Academy Of Sciences, Gedimino 3, LT-01103 Vilnius, Lithuania*
[2] *Kaunas University of Technology, Studentų 56-350, LT-51424 Kaunas, Lithuania*
[3] *Vytautas Magnus University, Vileikos 8-702, LT-44404 Kaunas, Lithuania*

Received 04 November 2009; accepted 30 April 2010

Two methods on the basis of time averaged projection moiré techniques are proposed for the identification of the machine and cross directions of paper and paperboard. For industrial applications the method based on the analysis of the experimental images of the three lowest eigenmodes is proposed. Orthotropic constitutive model is to be taken into account for machine made paper and single ply paperboard. It is assumed that the paper and paperboard in a printing device are loaded in their planes and thus a problem of plane stress is solved. Then the eigenmodes of bending vibrations as of a plate are analyzed.

Keywords: machine made paper, machine made paperboard, machine direction (MD), cross direction (CD), orthotropic model, finite elements, plane stress, plate bending, eigenmodes, time averaged projection moiré, experimental setup, methods of identification.

1. INTRODUCTION

The qualities of paper and paperboard in the longitudinal and transverse directions of production of the sheet differ, because in the process of production of the material most of the wooden plies are directed according to the machine direction of the paper [1]. Such initial distribution of plies in the material, the length of the plies being much greater than their diameter, influences the anisotropic mechanical properties of the material of the paper and paperboard (see Fig. 1). The anisotropy of the paper when compared with the paperboard is lower because of the lower thickness of the sheet of it [2]. In the research papers [3 – 5] after investigating the qualities of different types of paper and paperboard it was determined that the ratio of the qualities of the material in the machine direction (MD) and cross direction (CD) is about 1.2 ÷ 2. In the process of design and production of products made from paper and paperboard it is important to take into account anisotropic qualities of the material and to choose the machine direction of production of those materials in the correct way. In the paper [6] it is determined that the machine direction of paperboard has substantial influence on the qualities of resistance to statical compression of paperboard boxes. Different qualities of paper and paperboard in the machine and cross directions also influence the eigenmodes of their materials experiencing tension and action of vibrations. In the paper [7] the influence of defects in the thin sheet of paper to the eigenmodes was investigated. In the paper [8] according to the obtained low frequency modes of vibrations of the paperboard the modulus of elasticity and Poisson‘s ratios in the machine and cross directions of the paperboard were determined. The ratio of the determined parameters is about two. Orthotropic qualities of paper and paperboard are and can be investigated by other non-destructive methods such as ultrasound [9] and the method of projection moire [10].

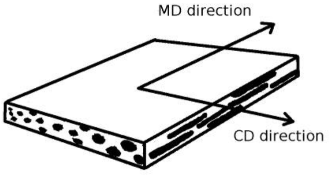


Fig. 1. The principal material directions for paper/paperboard: machine direction (MD), cross machine direction (CD)

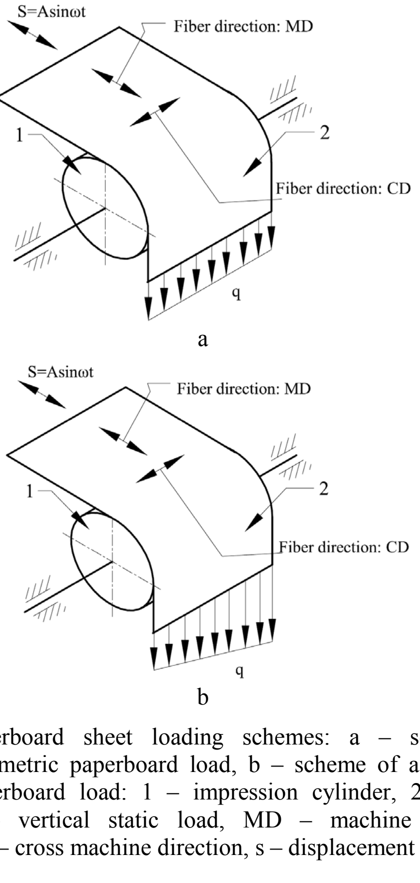


Fig. 2. Paperboard sheet loading schemes: a – scheme of symmetric paperboard load, b – scheme of asymmetric paperboard load: 1 – impression cylinder, 2 – paper, q – vertical static load, MD – machine direction, CD – cross machine direction, s – displacement

---

*Corresponding author. Tel.: +370-37-300237; fax.: +370-37-451684.
E-mail address: *edmundas.kibirkstis@ktu.lt* (E. Kibirkštis)

The purpose of this paper is to identify the machine direction of the paper and paperboard and also the character of tension of the sheet of those materials in the printing machine by using non-destructive methods on the basis of experimental optical methods of investigation.

The sheet of paper and paperboard in the printing machine may be loaded symmetrically (see Fig. 2, a) and non-symmetrically (see Fig. 2, b).

The model for the analysis of vibrations of machine made paperboard in a printing machine is proposed on the basis of the material described in [11, 12]. It is assumed that the paperboard in a printing machine is loaded in its plane. The static problem of plane stress by assuming the displacements at the boundary of the analyzed paperboard to be given is solved. Then the eigenmodes of bending vibrations as of a plate are calculated.

Two methods on the basis of time averaged projection moiré techniques are proposed for the identification of the machine and cross directions of a paper and paperboard.

Experimental investigations of the eigenmodes of vibrations of the paper and paperboard were performed using the experimental setup described in [12].

2. MODEL FOR THE ANALYSIS OF VIBRATIONS OF THE PAPER AND PAPERBOARD

Further x, y and z denote the axes of the system of coordinates. First the static problem of plane stress is analyzed. The element has two nodal degrees of freedom: the displacements u and v in the directions of the axes x and y. The stiffness matrix has the form:

$$[K]=\int[B]^T[D][B]h\,dxdy, \tag{1}$$

where h is the thickness of the paperboard and:

$$[B]=\begin{bmatrix}\frac{\partial N_1}{\partial x} & 0 & \dots\\ 0 & \frac{\partial N_1}{\partial y} & \dots\\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \dots\end{bmatrix}, \tag{2}$$

where N_i are the shape functions of the finite element and:

$$[D]=\begin{bmatrix}\frac{E_x}{1-\nu_{xy}\nu_{yx}} & \frac{E_y\nu_{xy}}{1-\nu_{xy}\nu_{yx}} & 0\\ \frac{E_x\nu_{yx}}{1-\nu_{xy}\nu_{yx}} & \frac{E_y}{1-\nu_{xy}\nu_{yx}} & 0\\ 0 & 0 & \frac{E_xE_y}{E_x+E_y+E_x\nu_{yx}+E_y\nu_{xy}}\end{bmatrix}, \tag{3}$$

where E_x and E_y are the modulus of elasticity and ν_{xy} and ν_{yx} are the Poisson's ratios. The vector of displacements $\{\delta\}$ is determined by solving the system of linear algebraic equations.

Then the eigenproblem of the plate is solved. The element has three nodal degrees of freedom: the transverse displacement of the paper w and the rotations Θ_x and Θ_y about the axes of coordinates x and y. The mass matrix has the form:

$$[\bar{M}]=\int[N]^T\begin{bmatrix}\rho h & 0 & 0\\ 0 & \frac{\rho h^3}{12} & 0\\ 0 & 0 & \frac{\rho h^3}{12}\end{bmatrix}[N]dxdy, \tag{4}$$

where ρ is the density of the material of the paper and:

$$[N]=\begin{bmatrix}N_1 & 0 & 0 & \dots\\ 0 & N_1 & 0 & \dots\\ 0 & 0 & N_1 & \dots\end{bmatrix}. \tag{5}$$

The stiffness matrix has the form:

$$[\bar{K}]=\int\begin{pmatrix}[\bar{B}]^T\frac{h^3}{12}[D][\bar{B}]+\\ +[\bar{\bar{B}}]^T\frac{E_xE_yh}{(E_x+E_y+E_x\nu_{yx}+E_y\nu_{xy})1.2}[\bar{\bar{B}}]+\\ +[G]^T[M_\sigma][G]\end{pmatrix}dxdy, \tag{6}$$

where:

$$[\bar{B}]=\begin{bmatrix}0 & 0 & \frac{\partial N_1}{\partial x} & \dots\\ 0 & -\frac{\partial N_1}{\partial y} & 0 & \dots\\ 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \dots\end{bmatrix}, \tag{7}$$

$$[\bar{\bar{B}}]=\begin{bmatrix}\frac{\partial N_1}{\partial y} & -N_1 & 0 & \dots\\ \frac{\partial N_1}{\partial x} & 0 & N_1 & \dots\end{bmatrix}, \tag{8}$$

$$[G]=\begin{bmatrix}\frac{\partial N_1}{\partial x} & 0 & 0 & \dots\\ \frac{\partial N_1}{\partial y} & 0 & 0 & \dots\end{bmatrix}, \tag{9}$$

$$[M_\sigma]=h\begin{bmatrix}\sigma_x & \tau_{xy}\\ \tau_{xy} & \sigma_y\end{bmatrix}, \tag{10}$$

where the stresses σ_x, σ_y, τ_{xy} are determined from:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=[D][B]\{\delta\}. \tag{11}$$

3. EIGENMODES OF THE MACHINE MADE PAPERBOARD IN A PRINTING MACHINE UNDER SYMMETRIC LOADING

The displacements at the boundary of the analyzed paperboard are given and they produce the loading vector. The square piece of paperboard is analyzed. The following boundary conditions of symmetric loading are assumed: on the lower boundary it is assumed that $u = v = 0$; on the upper boundary it is assumed that $u = 0$ and $v = 10^{-5}$ m. The vector of displacements is determined by solving the system of linear algebraic equations.

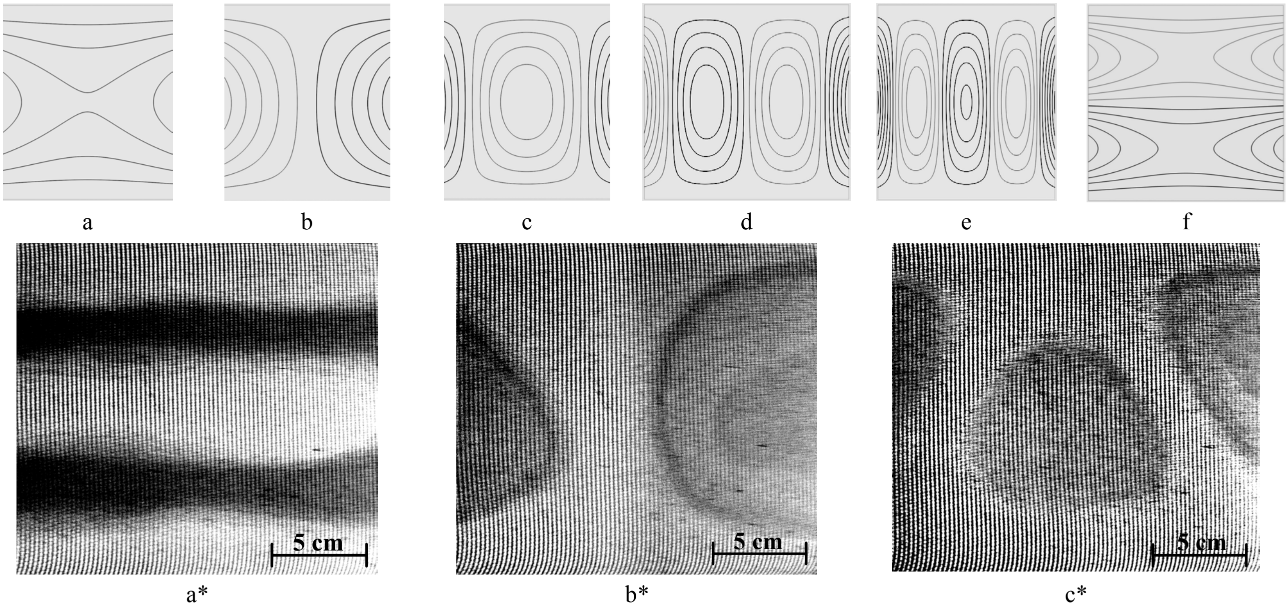
The main qualities of the material of the paper and paperboard used in the experimental investigations are presented in Table 1.

It is assumed that the modulus of elasticity $E_y = 0.68 \cdot 10^9$ Pa, Poisson's ratio $\nu_{xy} = 0.45$, Poisson's ratio $\nu_{yx} = 0.10$, thickness $h = 0.00027$ m, density $\rho = 1111$ kg/m^3. The eigenmodes for machine direction of paperboard are presented in Fig. 3, a – f). The eigenmodes of vibration of this paperboard obtained by using the method of experimental investigation are presented in Fig. 3, a* – c*.

For another problem it is assumed that the modulus of elasticity $E_y = 2.00 \cdot 10^9$ Pa, Poisson's ratio $\nu_{xy} = 0.10$, Poisson's ratio $\nu_{yx} = 0.45$. The eigenmodes for cross direction of paperboard are presented in Fig. 4, a – f. For this problem the results of experimental investigations are presented in Fig. 4, a* – c*.

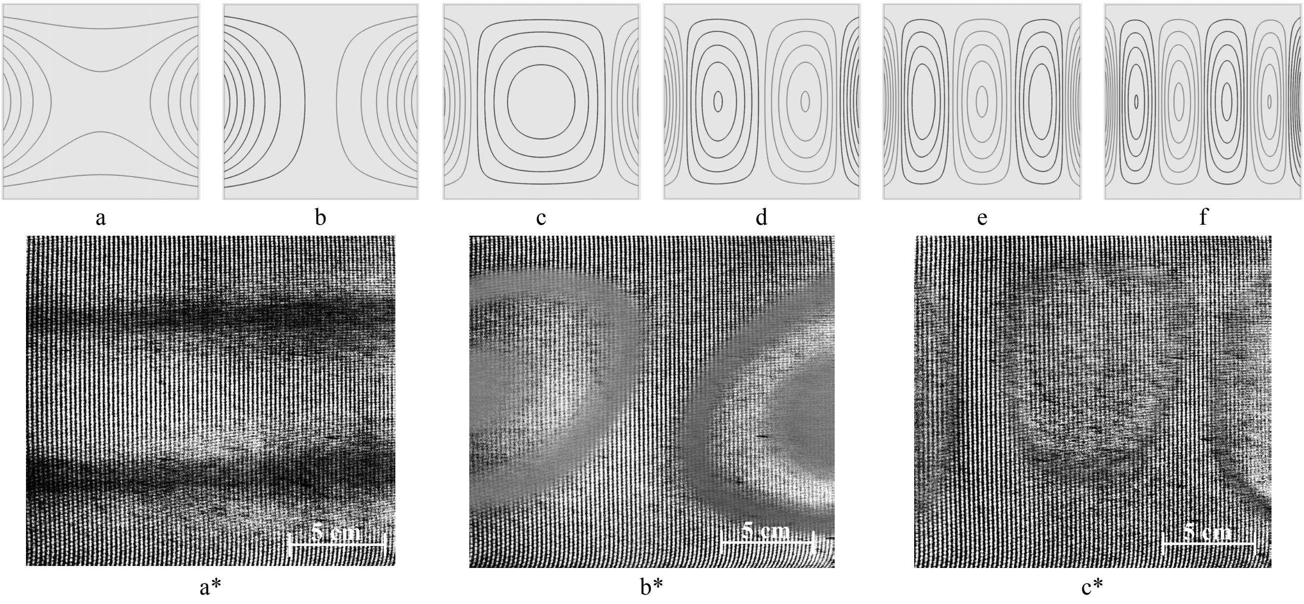
Table 1. Technical characteristics of paper and paperboard

Property	Paperboard	Paper
Basic weight, g/m^2	300	80
Paperboard sheet thickness, μm	270	102
Density, kg/m^3	1111	785
Paperboard stifness MD/CD, mNm	3.6/2.4	0.42/0.31
Poisson's ratio ν, MD/CD	0.45/0.10	0.40/0.14
Modulus of elasticity E, MD/CD GPa	2.00/0.68	1.10/0.34


a b c d e f

a* b* c*

Fig. 3. The eigenmodes for machine direction of paperboard under symmetric loading: a – first eigenmode, b – second eigenmode, c – third eigenmode, d – fourth eigenmode, e – fifth eigenmode, f – sixth eigenmode, a* – experimental projection moiré image of the first eigenmode, frequency – 122 Hz, b* – experimental projection moiré image of the second eigenmode, frequency – 128 Hz, c* – experimental projection moiré image of the third eigenmode, frequency – 145 Hz. Amplitude: 8×10^{-6} m


a b c d e f

a* b* c*

Fig. 4. The eigenmodes for cross direction of paperboard under symmetric loading: a – first eigenmode, b – second eigenmode, c – third eigenmode, d – fourth eigenmode, e – fifth eigenmode, f – sixth eigenmode, a* – experimental projection moiré image of the first eigenmode, frequency – 120 Hz, b* – experimental projection moiré image of the second eigenmode, frequency – 126 Hz, c* – experimental projection moiré image of the third eigenmode, frequency – 141 Hz. Amplitude: 8×10^{-6} m

4. EIGENMODES OF THE MACHINE MADE PAPER IN A PRINTING MACHINE UNDER UN-SYMMETRIC LOADING

For un-symmetric loading the boundary conditions on the upper boundary are changed, that is on the upper boundary linear variation of the displacement v is assumed with $v = 5\cdot10^{-6}$ m on the left side of the boundary and $v = 1.5\cdot10^{-5}$ m on the right side of the boundary.

The first eigenmodes for machine direction of the paper are presented in Fig. 5 (a – f).

For this problem the results of experimental investigation for the first, second and third eigenmode of vibrations are presented in Fig. 5 (a* – c*).

The first eigenmodes for cross direction of the paper are presented in Fig. 6 (a – f).

For this problem the results of experimental investigation for the first, second and third eigenmode of vibrations are presented in Fig. 6 (a* – c*).

From the obtained results (Fig. 3 – 6 (a – c and a* – c*)) is seen the satisfactory congruity of the theoretical and experimental results.

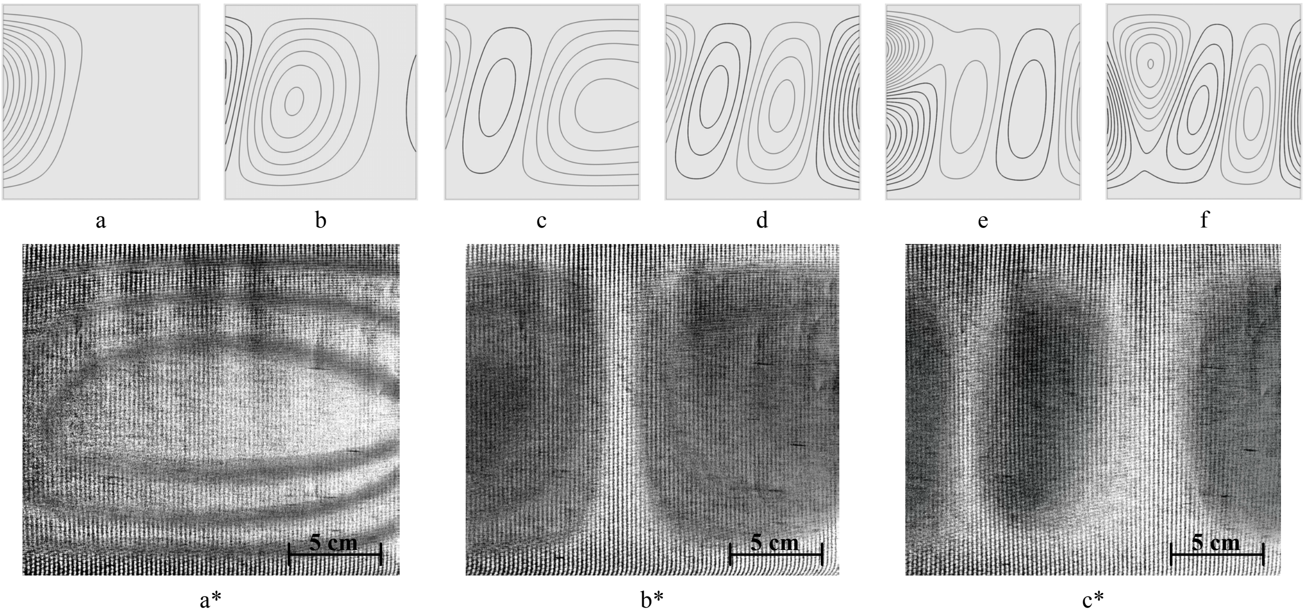


Fig. 5. The eigenmodes for machine direction of paper under un-symmetric loading: a – first eigenmode, b – second eigenmode, c – third eigenmode, d – fourth eigenmode, e – fifth eigenmode, f – sixth eigenmode, a* – experimental projection moiré image of the first eigenmode, frequency – 152 Hz, b* – experimental projection moiré image of the second eigenmode, frequency – 175 Hz, c* – experimental projection moiré image of the third eigenmode, frequency – 205 Hz. Amplitude: 4×10^{-6} m

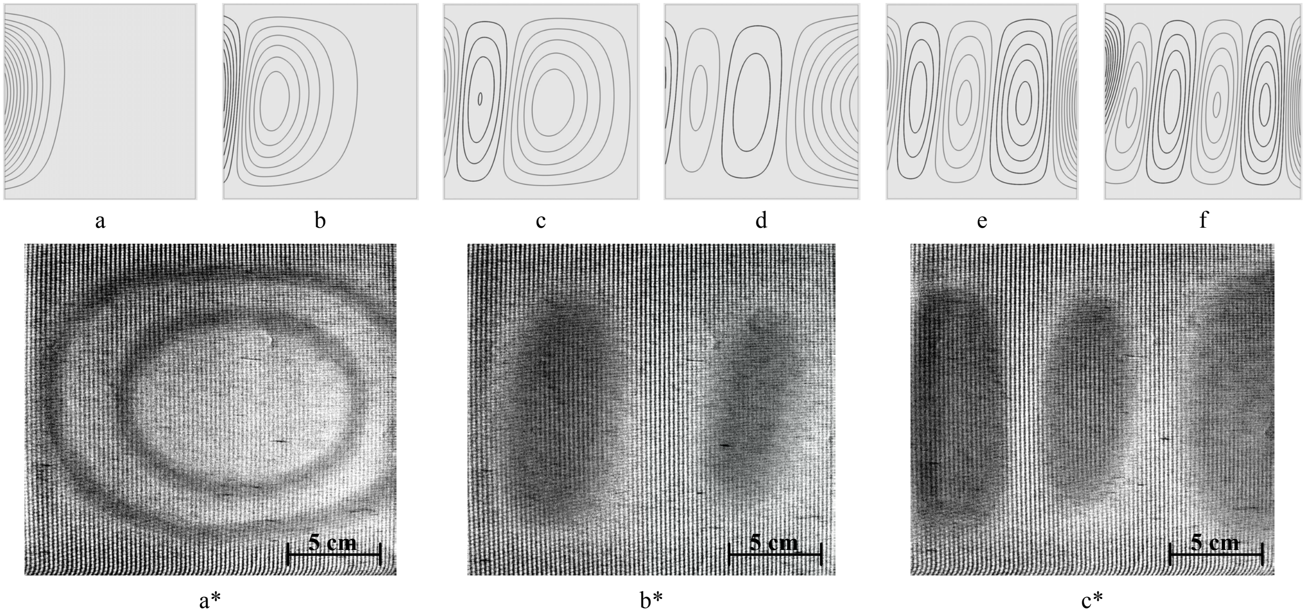


Fig. 6. The eigenmodes for cross direction of paper under un-symmetric loading: a – first eigenmode, b – second eigenmode, c – third eigenmode, d – fourth eigenmode, e – fifth eigenmode, f – sixth eigenmode, a* – experimental projection moiré image of the first eigenmode, frequency – 146 Hz, b* – experimental projection moiré image of the second eigenmode, frequency – 170 Hz, c* – experimental projection moiré image of the third eigenmode, frequency – 194 Hz. Amplitude: 4×10^{-6} m

5. NON-DESTRUCTIVE IDENTIFICATION OF THE MACHINE AND CROSS DIRECTIONS OF A PAPER AND PAPERBOARD

Two methods are proposed for the solution of this problem.

Method 1. This method is based on the analysis of distribution of deflection in the first eigenmode. From Fig. 3, a, and Fig. 4, a, it is evident that both eigenmodes are of similar character, but the distribution of deflections in them is different. Thus from the experimental images obtained by using time averaged projection moiré it is possible to identify the machine direction and the cross direction.

Practical application of the method showed that in industrial environments external noise and vibrations caused by the elements of the printing device and other sources influence the accuracy of measurement. Another source of inaccuracy is the difficulty of assuring precise symmetry of loading. Thus the method was generalised by analyzing not only the first eigenmodes, but several lowest eigenmodes. On the basis of practical experience it is recommended to analyze the time averaged projection moiré images of the three lowest eigenmodes. From the presented experimental results in the figures it is evident that the experimental images of the second eigenmodes enable to clearly identify the directions of orthotropy of the polygraphic material. When interpreting the experimental time averaged projection moiré images one is to have in mind that the loading is not always precisely symmetric. In order to be able to judge about the errors introduced by the non-symmetry of loading one is to have in mind the general character of the eigenmodes for non-symmetric loading of the polygraphic material which are presented in the figures. When the first eigenmodes become too similar to the ones under non-symmetric loading, one is to take efforts to increase the symmetricity of loading of the printing material in the printing device and only then the experimental measurements are to be continued.

Method 2. This method is based on the analysis of the lowest eigenmode which is of totally different character. From Fig. 3, f, and Fig. 4, f, it is evident that the sixth eigenmodes are of different character. Thus from the experimental images obtained by using time averaged projection moiré it is possible to identify the machine direction and the cross direction. In this method one is to be careful when calculating which eigenmode is being registered in the experiment.

Experimental investigations indicated that for higher eigenmodes the excited vibrations are usually of lower amplitude and thus require higher precision of measurement (higher density of moiré lines using time averaged projection moiré). This method is more sensitive to environmental noise and vibrations and can be implemented only in laboratory investigations. Thus for industrial applications the previous method is recommended. For the materials with higher anisotropy the eigenmodes of totally different character would be lower in their sequence and they would be among the first eigenmodes of the polygraphic material. In such problems this method would be advantageous with respect to the previous one. But for the usual polygraphic materials experimental investigations indicate that the anisotropy is insufficient for successful industrial application of this method.

6. CONCLUSIONS

The model for the analysis of vibrations of machine made paper and paperboard in a printing machine is proposed using the orthotropic constitutive relationship. It is assumed that a paper in a printing device is loaded in its plane. The static problem of plane stress by assuming the displacements at the boundary of the analyzed paper to be given is solved. Then the eigenmodes of bending vibrations as of a plate are calculated.

Symmetric and un-symmetric loading problems are analyzed. Both machine direction as well as cross direction are considered as directions of loading.

Two methods on the basis of time averaged projection moiré techniques are proposed for the identification of the machine and cross directions of polygraphic materials. For industrial applications the method based on the analysis of the experimental images of the three lowest eigenmodes is proposed.

Two methods for the identification of the directions of orthotrophy have been proposed:

1) on the basis of the image of the lowest eigenmode;
2) on the basis of the lowest eigenmode of substantially different character (in our problem for the sixth eigenmode).

Both methods were insufficiently reliable in industrial environments under the action of external noise and vibrations. So the first method was generalized: the images of several lowest eigenmodes were used in the procedure of identification of the directions of orthotrophy.

Extensive experimental investigations enabled to reach a conclusion that the use of the three lowest eigenmodes produces reliable results in industrial applications of the proposed method of identification. Thus the final recommendation for practical implementations of the method of identification is to register the images of the three lowest eigenmodes and to perform their mutual comparisons. Because of some errors in the symmetricity of loading and because of the action of external noise and vibrations the identification of the directions of orthotrophy may not be possible from the images of some of those eigenmodes, but among them there usually is at least one eigenmode from the images of which the identification can be successfully performed (in the presented results of experimental investigations it was the second eigenmode which evidently enabled to identify the directions of orthotrophy).

The obtained results are used in the process of identification of paper and paperboard material properties and the character of loading of the printing material in the printing machine.

REFERENCES

1. Paper and Board - Determination of Thickness, Density and Specific Volume. ISO 534:2005, 2005. International Standart.

2. **Qingxi, S. X., Boyce, M. C., Parks, D.** A Constitutive Model for the Anisotropic Elastic – Plastic Deformation of Paper and Paperboard *International Journal of Solids and Structures* 39 2002: pp. 4053 – 4071.
3. **Zhang, X., Jackson, T., Lafond, E.** Stiffness Properties and Stiffness Orientation Distributions for Various Paper Grades by Non-Contact Laser Ultrasonics *NDT&E International* 39 2006: pp. 594 – 601.
4. **Stenberg, N.** A Model for the Through – Thickness Elastic –Plastic Behaviour of Paper *International Journal of Solids and Structures* 40 (1) 2003: pp. 7483 – 7498.
5. **Aboura, Z., Talbi, N., Allaoui, S., Benzeggagh, M. L.** Elastic Behavior of Corrugated Cardboard: Experiments and Modeling. *Composite Structures*, 63, 2004: pp. 53 – 62.
6. **Kibirkštis E., Lebedys A., Kabelkaitė A., Havenko S.** Experimental Study of Paperboard Package Resistance to Compression *Mechanika* 1 (63) 2007: pp. 27 – 33.
7. **Kibirkštis, E., Kabelkaitė, A., Dabkevičius, A., Bivainis, V., Ragulskis, L.** Effect of Continuous Defects to the Vibrations of a Sheet of Paper *Ultragarsas* 64 (1) 2009: pp. 18 – 23.
8. **Sato, J., Hutchings, I. M., Woodhouse, J.** Determination of the Dynamic Elastic Properties of Paper and Paperboard from the Low - Frequency Vibration Modes of Rectangular Plates *Appita Journal: Journal of the Technical Association of the Australian and New Zealand Pulp and Paper Industry* 61 (4) 2007: pp. 291 – 296.
9. **Kažys, R., Stolpe, P.** Ultrasonic Non-Destructive On-Line Estimation of the Tensile Stiffness of a Running Paper Web *NDT&E International* 34 2001: pp. 259 – 267.
10. **Kibirkštis, E., Kabelkaitė, A., Dabkevičius, A., Bivainis, V., Ragulskis, L.** Non-Destructive Diagnostics of Uniformity of Loading of the Sheet of Paper *Ultragarsas* 64 (3) 2009: pp. 24 – 28.
11. **Castro, J., Ostoja-Starzewski, M.** Elasto – Plasticity of Paper *International Journal of Plasticity* 19 2003: pp. 2083 – 2098.
12. **Kabelkaitė, A., Kibirkštis, E., Ragulskis, L., Dabkevičius, A.** Analysis of Vibrations of Paper in a Printing Device *Journal of Vibroengineering* 9 (1) 2007: pp. 41 – 50.

DOI: 10.5755/j02.ms.26065