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A Non-Linear Deformation Model for Particle Impact Process

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Mathematical formulation of the particle impact processes is important in many different areas, such as the assessment of pneumatic transportation efficiency, the development theories of abrasive erosion and material shot peening, etc.

The aim of this paper is to analyse the possibilities of application of a non-linear normal deformation model to the particle impact process. Total and residual indentation depth and the normal force component applied to target material surface in the course of particle impact were calculated by the method above. Maximum pressure on target material, the radius of the impact crater and radial stresses were evaluated. Principal stresses under the impact crater were found.

Analytical results were compared with the experimental data, given in the Gommel's work, where the restitution coefficient, maximum pressure on target material surface and the residual depth of indentation produced by particle impact were practically measured. The spherical steel particles and the target material with different hardnesses of (285 – 790) VHN and (190 – 725) VHN respectively along with particle velocities of 3 m/s – 70.5 m/s and the impact angle of 90° were used in these experiments. It was shown that in case of soft particles the discrepancy between experimental and calculated results is rather significant. This can be explained by the presence of plastic deformations and respective increase of the contact area in real impacts, not considered in theory. The correlation between the experimental and calculated results is much better in the cases of harder particles. It has been shown that the non-linear normal deformation model is applicable for particle impact processes characterisation under different impact angles in the cases when plastic deformation are avoided during the impact.

Keywords: erosion, particle impact, contact pressure, contact deformations.

1. INTRODUCTION

Mathematical formulation of the impact process is important for many different domains of science. For example impact process formulation is necessary for the assessment of pneumatic transportation efficiency. At the same time this process formulation can be used for building up the theories of abrasive erosion and material shot peening. The aim of this paper is to analyse the possibilities of application of a non-linear normal deformation model to particle impact process. Applicability of the non-linear deformation model for the erosion wear of ceramics has been previously investigated and analysed in [1]. The present study widens the scope of this model.

2. THEORY

The first phase of particle impact, where the elastic and plastic deformations are activated, the dependence between normal force component F_n, generated by particle impact and the indentation depth in target material surface α are in following relation [1]:

$$F_n = b\alpha^s , \tag{1}$$

where b and s are the constants, which' values must be determined experimentally.

The second phase of particle impact follows the law of Gerstner [1]:

$$F_n = K \cdot (\alpha_{\max} - \alpha)^{3/2} , \tag{2}$$

where $\alpha_{\max}$ is the maximum indentation depth in target material surface, produced by particle impact and K is a constant, calculated with following equation:

$$K = \frac{4}{3}\sqrt{R}\left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)^{-1} , \tag{3}$$

where R is the particle radius, E_1 and μ_1 are the elastic modulus and Poisson's ratio of the particle, E_2 and μ_2 are the elastic modulus and Poisson's ratio of the target material.

Restitution coefficient for the normal velocity component [1]:

$$T_n = \frac{v_{2n}}{v_{1n}} = 5^{-1/2} K^{-1/3} [(mv_{1n}^2)^{2s-3} b^5 2^{s+6} (s+1)^{5s}]^{\frac{1}{6(s+1)}} , \tag{4}$$

where m is the particle mass, v_{1n} is the particle normal velocity component before the impact and v_{2n} is the particle normal velocity component after the impact.

The maximum indentation depth in target material surface:

$$\alpha_{\max} = \left[\frac{mv_{1n}^2 (s+1)}{2b}\right]^{\frac{1}{s+1}} \tag{5}$$

and taking in account equation (1), the maximum normal force component can be expressed as:

$$F_n^{\max} = \left[\frac{mv_{1n}^2 (s+1) \cdot b^{1/s}}{2}\right]^{\frac{s}{s+1}} . \tag{6}$$

The residual indentation depth is formulated as:

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$$\alpha_* = \alpha_{\max} - \left(b\alpha_{\max}^s / K\right)^{2/3}. \quad (7)$$

Maximum pressure on the target material surface, produced by particle impact can be expressed by the equation

$$q_{\max} = 0.615 F_{\max} \left[\frac{E_1 E_2}{R(E_1 + E_2)}\right]^{2/3} \quad (8)$$

and the radius of the impact crater follows the equation:

$$\rho = 0{,}881 \cdot \sqrt[3]{q_{\max} R \frac{(E_1 + E_2)}{E_1 E_2}}. \quad (9)$$

Radial stresses at the impact crater edge can be calculated using the formula:

$$\sigma_r = \frac{(1 - 2\mu_2)}{3} q_{\max}. \quad (10)$$

Principal stresses σ_x and σ_z under the impact crater can be determined by the relationship

$$\sigma_x = -q_{\max}\left[(1 - \mu_1)\left(1 - \frac{z}{\rho} arcctg \frac{z}{\rho} - \frac{1}{2}\frac{\rho^2}{\rho^2 + z^2}\right)\right] \quad (11)$$

and

$$\sigma_z = -q_{\max} \frac{\rho^2}{\rho^2 + z^2}, \quad (12)$$

where z is the co-ordinate according to the axis directed into the material, perpendicularly to the target surface.

3. RESULTS

There is not much experimental data on particle impact related deformation processes available in the literature. Gommel's thesis from 1967 [2] was considered the most suitable and was therefore utilised in first phase of present work in order to compare the model with the experimental results.

In Gommel's work the restitution coefficient, maximum pressure on target material surface and the residual depth of indentation produced by particle impact were measured. In his work Gommel used spherical steel particles (bearing balls) with different hardnesses: 285; 400; 560 and 790 VHN. The target material hardnesses were 190, 320, 440 and 725 VHN. The velocities of the particles before the impact were 3; 45.5; 58.3 and 70.5 m/s. The angle of impact 90° was used.

In our calculations constants b and s appearing in equation (1) were found with the use of stage-by-stage approximation method from equation (4) expressing the restitution coefficient for normal velocity component. By formula (6) there was calculated the maximum normal force component $F_{\max}$ and the residual depth of indentation by formula (7). The maximum values of the normal force components according to Gommel experiments and calculated by us are compared in Fig. 1.

Experimental values of the residual depth of indentation according to Gommel and calculated by us are presented in Fig. 2. In case of soft particles the difference between the experimental and calculated results is rather significant.

Differences between the calculated and experimental values that can be seen in Fig. 1 and Fig. 2, can be explained by the presence of plastic deformation of the particles occurring in the course of impact. This plastic deformation of particles is responsible for the increase of contact area.

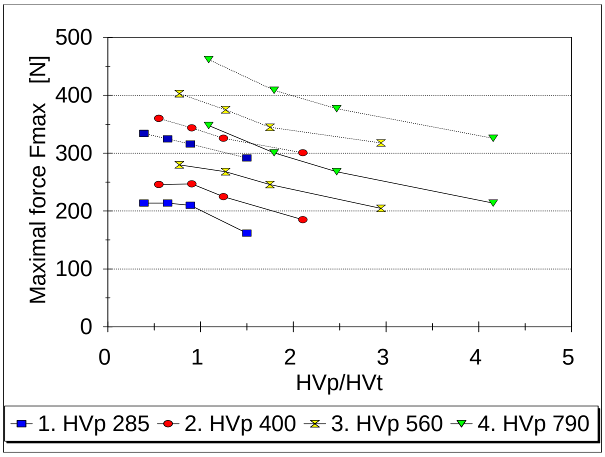

Fig. 1. The maximum normal force component F_{max} versus the ratio of particle and target material hardnesses HVp and HVt. Dashed lines show Gommel's experimental results [2] and solid lines express our calculations

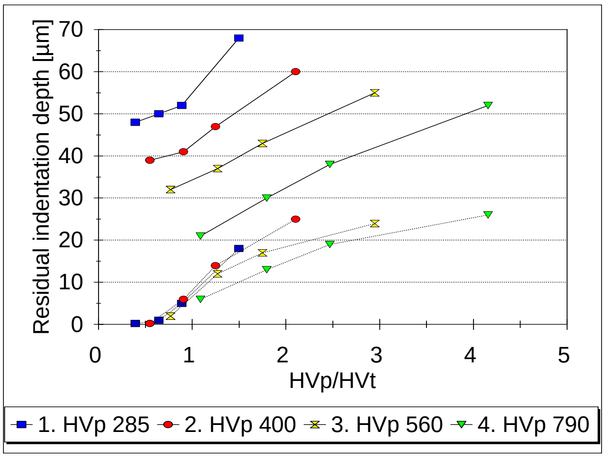

Fig. 2. Residual indentation depth versus the ratio of particle and target material hardness. Dashed lines show the Gommel's experimental results [2] and solid lines express our calculations

The correlation between the Gommel's experimental and our calculated values is better in the case of harder particles. With a decrease of particle hardness the difference between the calculated and experimental results will be more significant.

In order to avoid plastic deformation of particles during the impact process, glass beads with average diameter 1 mm were used in our experiments. For target materials glass plate and annealed steel 0.45 % C (200 VHN) were used. For particle velocity before impact 70 m/s chosen to avoid crushing of the particles. Impact angles 30°, 45° and 60° were employed.

Restitution coefficients for the normal velocity component T_n values were obtained experimentally, using a special Laser Doppler Anemometer [3], constants s and b being found from equation (4) of the restitution coefficient of the normal velocity component. Maximum pressure on the target material surface, produced by particle impact, was calculated by formula (8).

In case of annealed steel 0.45 % C as a target material, the principal stresses σ_x and σ_z under the impact crater were calculated by equations (11) and (12). The shearing stresses τ inside the target material were calculated as:

$$\tau = \frac{\sigma_x - \sigma_z}{2} \tag{13}$$

and the distance Δ inside the material was determined, where the shearing stresses will exceed the yield point R_e (i. e. work hardening of the material will take place).

Typical relation between the principal stresses and shearing stress versus depth of the point in the material is shown in Fig. 3. Distance Δ was also measured by X-ray method at St. Petersburg Technical University. Analytical results were compared with the experimental values obtained by X-ray method and the values of the maximum depth of work hardening Δ were found to be in rather good correlation. A comparison of the results is presented in Fig. 4.

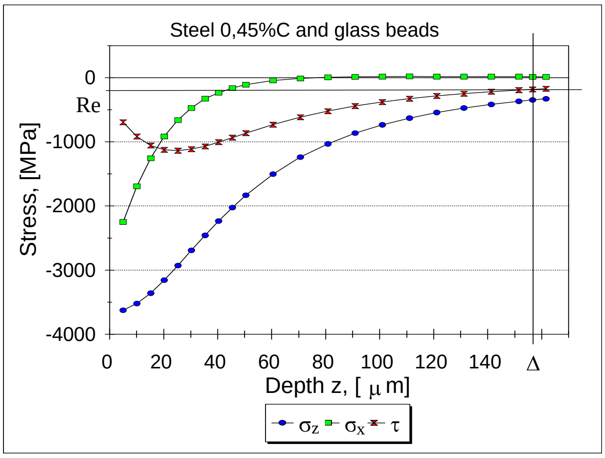

Fig. 3. Shearing stress τ and principal stresses σ_x and σ_z under the impact crater of glass particle according. Calculations were made for impact angle 30° and for impact velocity of 13.25 m/s

In case of impact of glass beads against a glass target, radius of impact crater and radial stresses at the impact crater edge were calculated using the equations (9) and (10) respectively. The calculated radii of impact crater were compared with radii of the circular cracks, measured under light microscope (Table 1). The correlation between the measured and calculated radii proved to be good.

Table 1. The measured and calculated radii of cracks

	Particle velocity, m/s	Crack radii on different impact angles		
		30°	45°	60°
Measured	32	69 ±8.5	89 ±4	94 ±4
Calculated		62	65	77
Measured	64	87 ±7	92.5 ±8	100 ±8.5
Calculated		80	84	99

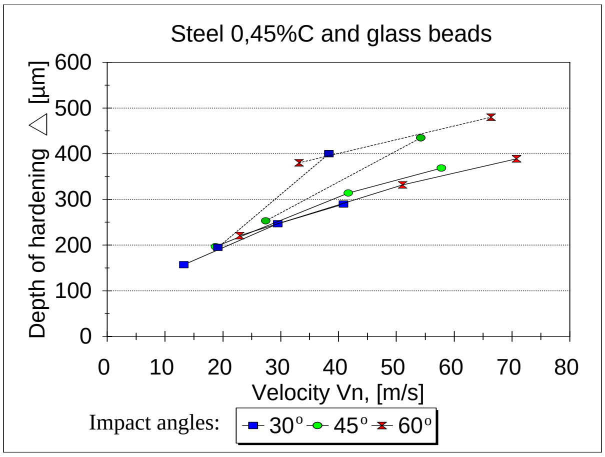

Fig. 4. Comparison of calculated and experimental results. Solid lines show the calculated results and dashed line shows the experimental results

4. CONCLUSION

The use of a non-linear deformation model in analytical treatment of particle impact process yields a satisfying correlation between the calculated and experimental results provided that the particle is not subject to plastic deformation during the impact.

REFERENCES

1. **Pappel, T., Järvpõld, N.** Adaption of a Non-linear Deformation Model for the Erosion Behaviour of Ceramics *Proceedings of the International Conference Balttrib'99*, Kaunas, 1999: pp. 274 – 280 (in Russian).
2. **Gommel, G.** Stoβuntersuchungen Stahlkugel-Stahlplatte im Zusammenhang mit Strahlmittelzertrümmerung und Strahlverschleiβ *Von der Technischen Hochschule Stuttgart zur Erlangung der Würde eines Doktor-Ingenieurs (Dr.-Ing.) genehmigte Abhandlung*, 1967.
3. **Kleis, I., Pappel, T., Hussainova, I., Štšeglov, I.** Investigation of Particle Impact Process *Journal of Friction and Wear* 18 (6) 1997: pp. 730 – 735.

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