

Linear-Regression-Equation-Based Prognosis of the Dependence of the Thickness of the Air Layer Equivalent to the Paint Coating's Water Vapour Permeability on the Type of the Paint and Number of the Paint Coating Layers

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For the wall's moisture state calculations it is important to know the values of the water vapour permeability parameters for the materials of which the wall layers are made. Therefore, if the exterior layer of the wall is painted repeatedly then the vapour resistance value of total paint layers do not conform to the declared values. Hence the aim of the paper is to determine the correlation between the number of the paint coating layers and water vapour resistance by presenting the linear regression equation that can be used to predict the value of the paint coating's vapour resistance factor in relation to the number of the paint coating layers. The results of calculation show, that the s_d value of the paint coating depends on the type of paint and the number of the paint coating layers n . The major inadequacy of the s_d value is determined for the acrylic paint.

Keywords: painted thin render, water vapour permeability, water vapour resistance factor, air layer thickness equivalent to water vapour permeability, moisture state.

INTRODUCTION

Recently, with the wide use of the new effective materials and products for the insulated buildings' finish, the defects caused by the condensed moisture accumulated in the envelopes became more frequent. This problem is especially acute for the process of designing the walls from outside insulated with the mineral wool slabs, which are finished with the painted thin render. Such construction is multilayer, and the value of water vapour permeability in each layer is different. When compared to the mineral wool's water vapour permeability, the exterior layer's (i. e. the paint coating's) water vapour permeability is rather low. Therefore the paint layer may start functioning as a barrier to water vapour diffusion and thus allow moisture accumulation in an envelope during the warm seasons [1–3]. Hence during the process of designing the buildings' envelopes the calculations of the wall's moisture state are carried out to determine the possibility for water vapour condensation and moisture accumulation under adequate environmental conditions.

Various calculation methods have been worked out to predict the wall's moisture state [4, 5]. Their reliability has been tested by experiments [6–9]. However, the major problem remains the determination of the values of the water vapour permeability parameters for the materials of which the wall layers are made that are necessary for such calculations. The problem is especially acute for the determination of the value of the paint coating's water vapour permeability. It should be stressed that the values of the paint coating's water vapour permeability declared by the producers are usually fixed when the paint is coated

on the base only once. However, the surface of the painted walls is affected by air pollution that causes soot, dust and salts to cover the surface of the finish. The paint components may react with the chemical materials existing in the air and water thereby forming new chemical combinations. Hence the exterior of the painted thin render undergoes alteration and the focuses of deterioration are formed. It should also be noted that, when affected by the solar ultraviolet radiation, the paint coating gets ageing, its color is fading and it starts cracking [10–13]. As a rule, to eliminate the exterior defects, the exterior surface is repainted yet without removing the old paint coating since its removal is problematic (i.e. thin render is mechanically easily destroyed). Therefore the properties of the paint coating's water vapour permeability alter.

The results of the experiments show that, with the increase of the number of the paint coating layers put on thin render, vapour resistance increases [14]. Therefore without the consideration of the number of the paint coating layers and an adequately increased vapour resistance the calculations of the wall's moisture state may be inaccurate. Hence the paper aims at the determination of the correlation between the number of the paint coating layers and water vapour resistance by working out the linear regression equation that will be used to predict the value of the paint coating's vapour resistance factor in relation to the number of the paint coating layers.

METHODOLOGY AND OBJECTIVES

The results of the paint coating's water permeability test presented in the [14] were used for the calculations. Table 1 gives the primary data of s_d (m) values for the air layer thickness equivalent to paint coating's water vapour permeability and the initial statistic assessment parameters. Figure 1 shows that the dependence of the air layer

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thickness s_d (m) equivalent to the paint coating's water vapour permeability on the number of its layers is linear.

The statistic significance of the s_d (m) value increase was estimated by applying the statistic One-way analysis of variance. The dependent variable is the thickness of the air layer s_d equivalent to the paint coating's water vapour permeability that depends on the following two factors (i.e. independent variables): the type of the paint coating (A) and the number of its layers (B).

Table 1. The s_d (m) values of the thickness of the air layer equivalent to the paint coating's water vapour permeability in relation to the type of the paint and number of its layers

Type of paint (A)	Number of the paint coating's layers n (B)				
	2	4	6	8	
Silicate 1	0.335	0.412	0.415	0.464	
Siloxan	0.163	0.169	0.194	0.225	
Acrylic 1	0.454	0.670	0.723	0.794	
Polyurethane	0.113	0.153	0.202	0.211	
Siloxan	0.160	0.195	0.233	0.276	
Silicate 2	0.101	0.120	0.133	0.155	
Acrylic2	0.435	0.474	0.635	0.740	
n_i	7	7	7	7	$n = 28$
$\sum x_i$	1.761	2.246	2.535	2.865	$\sum x_{ij} = 9.407$
\bar{x}_i	0.251	0.321	0.362	0.409	$\bar{x} = 0.336$
$\sum x_i^2$	0.583	1.022	1.245	1.589	$\sum x_{ij}^2 = 4.442$

n_i is the 'i'th' observation in a series of n observations; n – sample size; \bar{x}_i – sample mean of n 'i'th' values of x 'i'th'; \bar{x} – sample mean of n values of x ; $\sum x_i$ – the sum of x 'i'th'; $\sum x_i^2$ – the sum of squares of x 'i'th'; $\sum x_{ij}$ – the sum of x ; $\sum x_{ij}^2$ – the sum of squares of x .

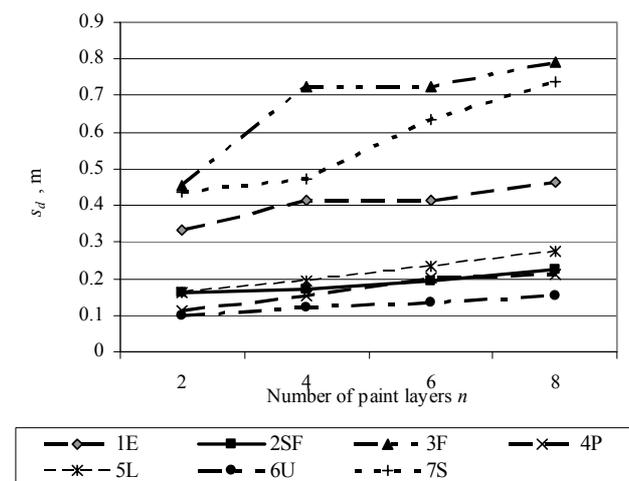


Fig. 1. Dependence of the thickness of the air layer s_d equivalent to the paint coating's water vapour permeability on the number of the paint layers. Paint marks: 1E – silicate 1; 2SF – siloxan; 3F – acrylic 1; 4P – polyurethane; 5L – siloxan-acrylic; 6U – silicate 2; 7S – acrylic 2

THE DEPENDENCE OF THE VALUE OF THE PAINT COATING'S WATER VAPOUR PERMEABILITY ON THE NUMBER OF ITS LAYERS

The verification of the assumption that the s_d value of the thickness of the air layer equivalent to the paint coating's water vapour permeability depends on the number of the paint layers was carried out in this research. With the choice of the level of significance making $\alpha = 0.05$, the statistical hypothesis is formulated as follows:

- $$\left\{ \begin{array}{l} H_0: \text{the means of the } s_d \text{ value in all the tested paint coatings do not differ due to the varying number of the paint layers;} \\ H_1: \text{the means of the } s_d \text{ value in at least two tested paint coatings differ due to the varying number of the paint layers.} \end{array} \right.$$

The null hypothesis H_0 means that the influence of the factor does not exist, i.e. the type of the paint coating does not affect the s_d value of the paint coating. The hypothesis H_0 is rejected (at least two means differ statistically significantly), if $F > F_\alpha(k-1, N-k)$; here $N = n_1 + \dots + n_k$, $F_\alpha(k-1, N-k)$ is the critical value of the significance level α when the Fisher distribution of the degrees of freedom is assumed to be $k-1$ and $N-k$. The hypothesis H_0 is not rejected if $F \leq F_\alpha(k-1, N-k)$. The following statistic is used to verify the ANOVA hypothesis:

$$F = \frac{\sum_{i=1}^k n_i (X_i - \bar{X})^2}{k-1} \cdot \frac{N-k}{\sum_{i=1}^k \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2} \quad (1)$$

Table 2 presents the p -value of the F statistics. It is lower than the level of significance α ($0.048 < 0.05$), therefore the hypothesis about the equal means is rejected. The means of the s_d parameter differ statistically significantly. Hence the s_d value of the paint coating is dependent on the number of the paint layers.

Table 2. The statistic ANOVA results of the increase of the paint coating's s_d value in relation to the number of layers

	Sum of squares	Degrees of freedom	Estimators of distribution	Statistic	p -value
Group	0.36	3	0.120	2.355	0.048
Interior	4.081	80	0.051		
Total	4.442	83			

Group – the differences of sample means; interior – the difference in the interior of the samples; total – the all sum of squares.

DEPENDENCE OF THE VALUE OF THE PAINT COATING'S WATER VAPOUR PERMEABILITY ON THE PAINT TYPE

The verification of the assumption that the s_d value of the thickness of the air layer equivalent to the paint coating's water vapour permeability depends on the type of paint was carried out. With the choice of the level of significance making $\alpha = 0.05$, the statistical hypothesis is formulated as follows:

H_0 : the means of the s_d value in all the tested paint coatings do not differ due to the type of paint;
 H_1 : the means of the s_d value in at least two tested paint coatings differ due to the varying types of paint.

Table 3 presents the p -value of the F statistics. It is lower than the level of α ($0.00 < 0.05$), therefore the hypothesis about the equal means is rejected. The means of the s_d parameter differ statistically significantly. Hence the s_d value of the paint coating is dependent on the type of paint.

Table 3. The statistic ANOVA results of the increase of the paint coating's s_d value in relation to its type

	Sum of squares	Degrees of freedom	Estimators of distribution	Statistic	p -value
Group	3.076	3	1.025	60.067	0.00
Interior	1.366	80	0.017		
Total	4.442	83			

Group – the differences of sample means; interior – the difference in the interior of the samples; total – the oll sum of squares.

By applying the one-way analysis of variance it is possible to find the answer to the question whether the means of the different samples are statistically significant. However, such analysis does not reveal what sample means differ statistically significantly. It may be determined by applying the post hoc criteria. The Tukey test is one of the most frequently applied post hoc criteria [15]. This test is based on the so-called distance Q of Student's statistic. The means \bar{x}_i and \bar{x}_j differ statistically significantly, if $|\bar{x}_i - \bar{x}_j| > TSD$; here:

$$TSD = \sqrt{\frac{MSW}{n}} Q_{\alpha}(nk - k, k); \quad (2)$$

$$MSW = \frac{\sum_{i=1}^k \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2}{N - k}. \quad (3)$$

Table 4. Results of the Tukey test

(I) Type of paint	(J) Type of paint	Value difference (I-J)	Standard deviation	p -value
Silicate	Siloxan	0.07	0.038	0.250
	Acrylic	-0.38	0.038	0.000
	Polyurethane	0.06	0.046	0.555
Siloxan	Silicate	-0.07	0.038	0.250
	Acrylic	-0.45	0.038	0.000
	Polyurethane	-0.0095	0.046	0.997
Acrylic	Silicate	0.38	0.038	0.000
	Siloxan	0.45	0.038	0.000
	Polyurethane	0.44	0.046	0.000
Polyurethane	Silicate	-0.061	0.046	0.555
	Siloxan	0.0096	0.046	0.997
	Acrylic	-0.44	0.046	0.000

Table 4 presents the p -value of the F statistics for the different paint types when the level of significance is $\alpha = 0.05$. The p -value of the acrylic paint is lower than the determined level of significance 0.05. Therefore the hypothesis about the equal means is rejected. The mean of the s_d value of the acrylic paint differs statistically significantly from the means of the s_d values of other paint coatings used in the test. The difference of the siloxan, polyurethane and silicate paint coatings is statistically insignificant. The statistic estimation of the dependence of the thickness of the air layer s_d equivalent to water vapour permeability on the type of paint is graphically presented in the box and whiskers plot (Fig. 2).

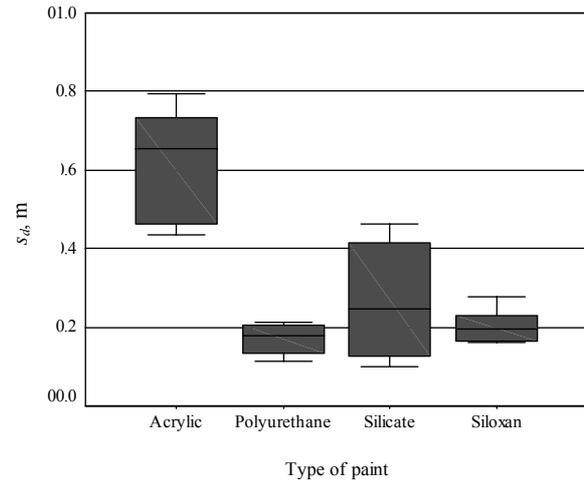


Fig. 2. The dependence of the thickness of the air layer s_d equivalent to the paint coating's water vapour permeability on the type of paint coating

Figure 2 shows that the mean of the s_d value of the thickness of the air layer equivalent to the paint coating's water vapour permeability is 0.639 m, the sample median is 0.67 m, the maximum value is 0.79 m, the lower quintile is 0.474 m and the upper quintile is 0.74 m. These values are the highest; therefore the acrylic paint coating may have an impact on the moisture state of the building walls. Furthermore, in order to use the reliable vapour resistances of the paint coating dependent on the number of paint layers for the calculation of the wall's moisture state, the linear regression equation will be worked out in the following section.

THE LINEAR REGRESSION EQUATION

The results of the testing of the paint coating's water vapour permeability demonstrate, that the dependence of the s_d value of the thickness of the air layer equivalent to the acrylic paint coating's water vapour permeability on the number of the paint coating layers is linear. It should be noted that the distribution of values around the means is homogeneous, i.e. it satisfies the homoscedasticity requirements (Fig. 3). Hence the s_d value of the acrylic paint may be prognoses on the basis of the probability equation of linear regression.

It is assumed that the number of the paint coating layers n is an independent variable (X). On the basis of the values of this variable the thickness of the air layer s_d equivalent to the acrylic paint coating's water vapour

permeability will be predicted and regarded as a dependent variable (Y). These variables are related by one-sided dependence. The mathematical means of the variables are presented in Table 5.

Table 5. Mathematical means of the variables

\bar{x}	\bar{y}	$\sum_{i=1}^8 y_i$	$\sum_{i=1}^8 x_i$	$\sum_{i=1}^8 x_i y_i$	$\sum_{i=1}^8 x_i^2$
5	0,616	4,925	40	26,774	240

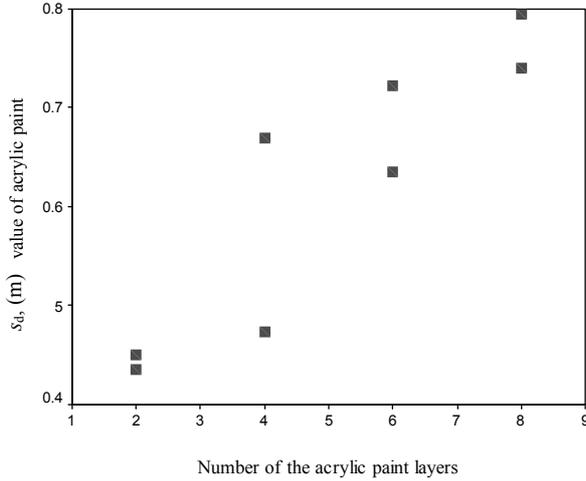


Fig. 3 Dependence of the thickness of the air layer s_d equivalent to the acrylic paint coating's water vapour permeability on the number of paint layers

Thus the most general linear probability model relating the interval variables X and Y is as follows:

$$Y = a + bX, \quad (4)$$

where: a, b are unknown coefficients. For the verification of the reliability of the model the estimators of these coefficients \hat{a} and \hat{b} are found out.

To select the estimators \hat{a} and \hat{b} of the parameters a and b , the method of the Smallest Squares was worked out. The estimators \hat{a} and \hat{b} are found out by minimization. Consider:

$$SSE = \sum_{i=1}^n (y_i - a - bx_i)^2. \quad (5)$$

The SSE sum is minimized as follows:

$$\hat{b} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = 0.054; \quad (6)$$

$$\hat{a} = \bar{y} - \hat{b}\bar{x} = 0.347. \quad (7)$$

The dependence of the s_d value of the thickness of the air layer equivalent to the acrylic paint coating's water vapour permeability on the number of the paint layers is expressed by the linear regression equation in the following way:

$$s_d = 0.347 + 0.054n. \quad (8)$$

Where: $s_d(m)$ is the thickness of the air layer equivalent to the acrylic painting's water vapour permeability; n is the number of the acrylic paint layers.

The average square error of the obtained $s_d - n$ dependence is found out as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{(n-2)}} = 0.023. \quad (9)$$

Furthermore, the verification of the reliability of the chosen model of regression is carried out. It requires the graphic analysis of residual, the finding of the coefficient of determination and the solution of the statistic hypothesis about the multiplier of the independent variable.

Figure 4 presents the diagram of the distribution of the standardized residuals, which demonstrates that the residuals \hat{e}_i of the double observation (x_i, y_i) are not high and are more or less equally distributed around the straight line $y = 0$. The observed y_i value shows an inconsiderable difference from the value that is obtained by the linear-regression-equation-based prognosis. It may be claimed that the application of the model of the linear regression fits for these data.

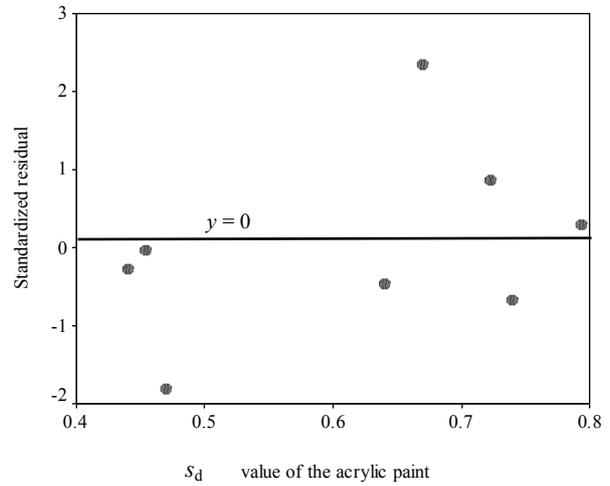


Fig. 4. Diagram of the distribution of standardized residuals

The normality of the observations $X \in N(x_m; \sigma^2)$ was estimated on the basis of the standardized residual histogram (Fig. 5), where N is the symbol showing normal distribution. The mathematical mean is equal to zero, and the dispersion makes 0.93. It leads to the conclusion that the values of the standardized residual are close to zero, i. e. the probability of this value is the highest.

After having calculated the coefficient of determination the value $r^2 = 0.817$ was obtained. It shows that the observations are considerably concentrated around the straight worked out by applying the method of the smallest squares. The model of regression explains 81.7 % of the variation. Therefore it may be claimed that the model of the linear regression equation fits to the obtained data.

The verification of the multiplier of the dependent variable is carried out by applying the method of the statistic hypothesis. Firstly, the prediction intervals for the coefficients a, b and dispersion σ^2 are to be found out as follows:

$$\hat{a} \pm s_a t_{(1-Q)/2}(n-2), \quad \hat{b} \pm s_b t_{(1-Q)/2}(n-2), \quad (10)$$

$$\left(\frac{SSE}{\chi_{(1-Q)/2}^2(n-2)}; \frac{SSE}{\chi_{(1+Q)/2}^2(n-2)} \right), \quad (11)$$

here: Q is the level of reliability; $t_{(1-Q)/2}(n-2)$ is the critical value of the $(1-Q)/2$ level of the Student distribution with $(n-2)$ degrees of freedom; $\chi_{(1+Q)/2}^2(n-2)$ is the critical value of the $(1+Q)/2$ level of the χ^2 distribution with $(n-2)$ degrees of freedom.

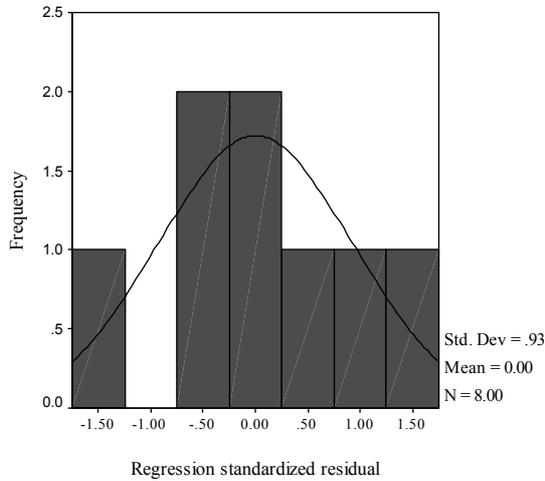


Fig. 5. The function $f(e)$ of the distribution density of standardized residual probability

The SSE is calculated according to (5). The estimator of dispersion is found out in the following way:

$$MSE = SSE / (n - 2). \quad (12)$$

The means of the coefficients a and b are calculated as follows:

$$s_a^2 = MSE \left(\frac{1}{n} + \frac{(\bar{x})^2}{(n-1)s_x^2} \right), \quad s_b^2 = \frac{MSE}{(n-1)s_x^2}; \quad (13)$$

here: $(n-1)s_x^2 = \sum_1^n x_i^2 - \left(\sum_1^n x_i \right)^2 / n$; s_x^2 is empirical x_1, \dots, x_n dispersion.

The probability is $Q = P\{b < b_{0.05}\} = p$. It has been determined that: $t_{0.025}(6) = 2.447$, $\chi_{0.025}^2(6) = 14.449$, $\chi_{0.975}^2(6) = 1.237$.

When the distributions of the estimators of the coefficients are known, it is possible to verify the statistic hypotheses about the values of the coefficients. It is important to find out whether coefficient b is not equal to zero since, in such a case, the model of regression is $Y_i = a + e_i$. In other words, Y_i does not depend on x_i . Thus the hypothesis that coefficient b is equal to zero is to be verified.

$$\begin{cases} H_0: b = 0, \text{ i. e. whether } s_d \text{ depends on } n; \\ H_1: b \neq 0, \text{ i. e. whether } s_d \text{ does not depend on } n. \end{cases}$$

The statistic $T = \hat{b} / s_b$ is calculated. It is assumed that the level of significance $\alpha = 0.05$. The hypothesis H_0 is rejected, if $|T| > t_{\alpha/2}(n-2)$; here $t_{\alpha/2}(n-2)$ is the critical value of the $\alpha/2$ level of the Student distribution with $(n-2)$ degrees of freedom. The hypothesis is rejected, if $|T| \leq t_{\alpha/2}(n-2)$. The results of the calculation are presented in Table 6.

Table 6. Parameters of the regression coefficients

Q	SSE	MSE	s_a	s_b	p -value
0.95	0.026	0.004	0.057	0.01	0.02

Table 6 shows that the p -value = 0.02, which is lower than the predicted level of significance $0.02 < 0.05$. The null hypothesis H_0 is rejected. Hence the number of the paint layers is significant for the prognosis of the thickness of the air layer s_d equivalent to the acrylic paint coating's water vapour permeability and should be included into the model of regression.

The 95 % prediction interval of the average value of the acrylic paint's s_d gauge, when the number of the acrylic paint layers is known, is as follows:

$$\left[\hat{y}(x) - \sqrt{SEY} t_{0.025}(n-2), \hat{y}(x) + \sqrt{SEY} t_{0.025}(n-2) \right]. \quad (14)$$

Thereby the estimator of the $EY - \hat{y}(x)$ dispersion is:

$$SEY = MSE \left(\frac{1}{n} + \frac{(x - \bar{x})^2}{(n-1)s_x^2} \right). \quad (15)$$

The results of the 95 % prediction interval of the average value of the acrylic paint's s_d gauge according to the linear regression equation are presented in Figure 6.

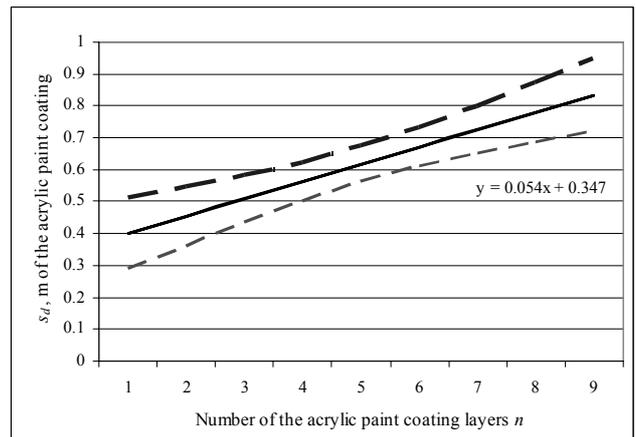


Fig. 6. The reliable interval of the dependence of the thickness of the air layer s_d equivalent to the acrylic paint coating's water vapour permeability on the mean of the number of paint layers

CONCLUSIONS

1. On the basis of the statistic One-way analysis of variance it has been determined that the s_d value of the paint coating depends on the type of paint and the number of the paint coating layers n .

2. The mean of the acrylic paint's s_d value differs statistically significantly from the means of the s_d values of other paint coatings used in the experiment. The difference of the means of the s_d values of the siloxan, polyurethane and silicate paint coatings is statistically insignificant.

3. The number of the paint layers n is significant for the prognosis of the thickness of the air layer s_d equivalent to the acrylic paint coating's water vapour permeability and should be included into the model of regression.

4. The linear regression equation has been worked out according to which it is possible to prognose the value of

the paint coating's vapour resistance in relation to the number of the paint coating layers.

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