

## Optimisation of the Aggregates Composition in Concrete

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Hardened concrete has a complex structure in which two parts can be distinguished – the matrix (hardened cement-sand grout) and the insertion of coarse aggregate particles. The aggregates make up to 80 % by volume of concrete. The cost of aggregates also reaches 30 % – 50 % of that of concrete. Therefore it is natural that aggregates largely affect the structure, physical-mechanical properties and price of concrete. In the present paper, the authors propose a mathematical method to design the grading of concrete aggregates. This method could provide a possibility to obtain more precise design of aggregates composition. This method is based on limiting by curves the maximum and minimum quantities of a mineral part of concrete mixture that can be passed through particular sieves.

*Keywords:* concrete, aggregate, limit curves, compressive strength, mathematical analogue, density.

### INTRODUCTION

One of the major goals of a concrete designer is to obtain such a ratio of mineral particles, cement and pores that corresponds to the best physico-mechanical properties of concrete. It is a complicated problem that has not been solved yet. The review of literature [1 – 6] has shown that at present most of studies are still based on conventional methods of concrete composition design, this means, that the aggregates of concrete are designed as two components (coarse aggregate and fine aggregate) mixture (ordinary method).

Precise design of aggregates composition was hampered because of high computational load and the lack of universal methods of calculations. The situation has dramatically changed when mathematical methods were proposed [7 – 12].

The packing of an aggregate for concrete is the degree of how good the solid particles of the aggregate would fill up the volume of the concrete. It is usually measured in terms of “packing density”, which is defined as the ratio of the solid volume of the aggregate particles to the bulk volume occupied by the aggregate [13].

From the packing density, the “voids ratio”, that is, the ratio of the volume of voids between the aggregate particles to the bulk volume occupied by the aggregate [13].

Depending on the size distribution and shape characteristics of the aggregate, the packing density may vary from 55 % to 85 %, while the corresponding voids ratio may vary from 45 % to 15 %.

Since the voids between the aggregate particles must be filled up with cement paste, the voids ratio determines the minimum volume of cement paste needed to produce concrete using the aggregate. A higher packing density leads to a smaller voids ratio and thus a smaller amount of cement paste is needed [13].

Packing of the aggregate used has a significant effect on the workability or the water requirement of the concrete mix produced [13].

Kaplan has shown that for a given mix proportion, the workability of the concrete mix decreases as the voids ratio increases [14].

Bloem and Gaynor have found that the water requirement of a concrete mix increases more or less linearly with the voids ratio of the aggregate used [15]. These effects may be explained by the fact that the cement paste added must first fill up the voids between the aggregate particles and it is the cement paste in excess of the amount needed to fill the voids that lubricates the concrete mix and gives the mix workability. Another possible reason is that some of the shape parameters affecting the packing of the aggregate such as the angularity of the aggregate particles may also be affecting the frictional forces between the particles and hence the workability of the concrete mix [13].

Packing of an aggregate has an indirect effect on the strength of the concrete made with the aggregate. If the packing is good, the water requirement will be smaller and a lower water-cement ratio may be used to produce a higher strength concrete [13]. For design and research of concrete, methods that examine concrete as a collection of bond particles are promising. Those methods can be used for improvement of concrete properties and solutions of concrete costs minimization or other optimisation problems.

In this paper, the aggregate (mineral material) is analysed as a collection of particles of particular shapes and sizes. Method of limit curves is used for modelling [9 – 12].

### MATHEMATICAL MODEL

The mathematical model of the method proposed is as follows:

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$$\left. \begin{aligned}
& \sum_{j=1}^m c_j x_j \rightarrow \min, & (a) \\
& b_{\min,i} \leq \sum_{j=1}^m a_{ij} x_j \leq b_{\max,i}, \quad i = 1, 2, \dots, n, & (b) \\
& \sum_{j=1}^m d_{vj} x_j \leq h_v, \quad v = 1, 2, \dots, s, & (c) \\
& \sum_{j=1}^m x_j = 1, & (d) \\
& x_j \geq 0. & (e)
\end{aligned} \right\} (1)$$

There  $a_{ij}$  is the quantity of mineral material  $j$  in per cent that passes through the sieve  $i$ ;  $b_{\min,i}$ ,  $b_{\max,i}$  are limited quantities of a mineral part of concrete mixture in per cent that can be passed through the sieve  $i$ ;  $n$  is the number of sieves;  $d_{vj}$  is the coefficient of additional inequality  $v$  corresponding to the mineral material  $j$ ;  $h_v$  is the limit value of additional inequality  $v$ ;  $s$  is the number of additional inequalities;  $m$  is the number of components;  $c_j$  is the weight multiplier of the component  $j$  (for example, the price of one mass unit of the component  $j$ );  $x_j$  is the quantity of the component (mineral material)  $j$  in parts of the unit.

The objective function (1, a) is commonly aimed to minimize the cost of mass unit of aggregate, the inequality (1, b) is limiting the grading of aggregate composition, the inequality (1, c) mathematically expresses technological limitations (for example, these inequalities can be used for limiting the maximum or minimum quantities of the particular mineral material in a mixture), the equation (1, d) shows, that the variables are expressed in parts of the unit and the inequality (1, e) ensures the reality of solution.

Thus, having solved the above problem, the lowest price aggregate composition is obtained. The second step of design is to find a composition of aggregate possessing the best physico-mechanical properties. In theory, it can be done when the points  $b_i$  ( $b_i = \sum_{j=1}^m a_{ij} x_j, i = 1, 2, \dots, n$ ) of the aggregate grading curve match the middle points  $b_{m,i}$  ( $b_{m,i} = \frac{b_{\min,i} + b_{\max,i}}{2}, i = 1, 2, \dots, n$ ) between the limit curves. To achieve this purpose, the interval between the limit curves should be narrowed step by step, with the problem (1) solved at every step.

At every step, the aggregate composition that is result of solution can be compared with "ideal" aggregate composition using a correlation coefficient:

$$\rho = \frac{\sum_{i=1}^n (b - \bar{b})(b_{m,i} - \bar{b}_m)}{\sqrt{\sum_{i=1}^n (b_i - \bar{b})^2 \sum_{i=1}^n (b_{m,i} - \bar{b}_m)^2}}, \quad (2)$$

$$\text{where } \bar{b} = \frac{1}{n} \sum_{i=1}^n b_i, \bar{b}_m = \frac{1}{n} \sum_{i=1}^n b_{m,i}.$$

The interval between limits curves can be narrowed in different ways. The authors are recommending to do it by using following equations:

$$\left. \begin{aligned}
& b_{\min,i} = b_{\min,i} + \frac{(b_{\max,i} - b_{\min,i})(k-1)}{2r}, \\
& b_{\max,i} = b_{\max,i} - \frac{(b_{\max,i} - b_{\min,i})(k-1)}{2r},
\end{aligned} \right\} (3)$$

Where  $b_{\min,i}$  and  $b_{\max,i}$  are the limits of percentage of particular aggregate components (fractions) on step  $k$ .

For the purpose to resave fine-grained or coarse-grained aggregate composition it is recommended to change points of minimum or maximum limit curve only. In all cases, process of narrowing of interval between limits curves is finished when the analogue (1) on routine step has no solution.

Sometimes, the analogue (1) has no solution at the first step. Then, it is recommended to find a fictitious solution, i.e. a solution not meeting the requirements. It means that the interval between the limit curves should be increased until a is found. The fictitious solution can be helpful for determining the reasons why a normal solution can not be found.

The proposed method, compared to commonly used approaches, has a number of advantages:

a) it allows us to define instantly the planned grading of a mineral part of the concrete from the mineral materials available;

b) it enables us to get the optimal mineral part of the concrete, according to a desired property of concrete, for example, the cheapest concrete or the highest density of concrete;

c) it makes possible get and analyse fictitious (not standard) mineral material composition.

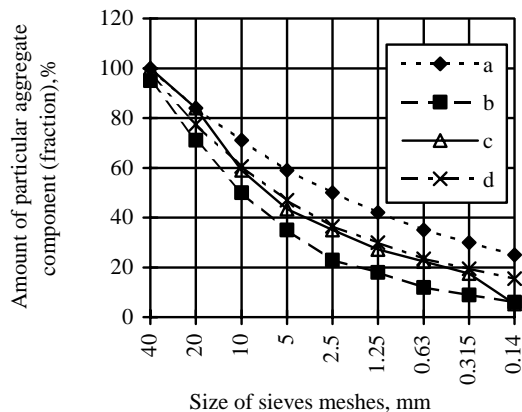
## THE EXPERIMENT

The packing of an aggregate is dependent on the size distribution of the aggregate particles. An aggregate having a good grading, in the sense that it contains particles of a wide range of size with the voids between particles filled by successively smaller particles, would generally have a relatively high packing density. The maximum density theory calls for an ideal grading curve, which is parabolic in shape when plotted to a natural scale. However, such an "ideal" grading curve, though capable of minimising the amount of cement paste needed, produces a harsh concrete mix. In actual practice, the addition of more than enough cement paste to fill the voids of the fine aggregate and more than enough mortar (cement paste, fine aggregate) to fill the voids of the coarse aggregate would produce a more workable concrete mix [1].

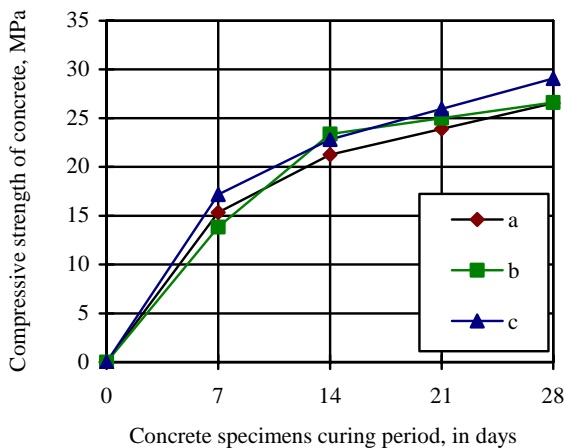
Limit curves were chosen with the aim to obtain the highest density aggregate composition (Fig. 1).

The comparative analysis of concrete compressive strength values was made. For this purpose, three sets of specimens were prepared with the same water-cement ratio (0.67) and the slump of concrete mix (2 cm – 4 cm). The specimens of the first set were designed using the ordinary method, when the aggregate composition is made out of two components: coarse aggregate (gravel) and fine aggregate (sand). The specimens of the second set were designed using three component aggregate composition (sand, gravel and crumble stone). The specimens of third series (Fig. 1, c) were designed using separated fractions of

the same components. The results of concrete compressive strength test showed that the compressive strength and hardening dynamics are similar for all sets of specimens. The data of testing are presented in Fig. 2.



**Fig. 1.** Grading of concrete aggregates composition: a – maximum limit curve of aggregates grading, b – minimum limit curve of aggregates grading, c – a curve of aggregate composition designed using a mathematical model (1), d – a curve of theoretically “ideal” aggregate composition



**Fig. 2.** Results of comparative analysis of concrete compressive strength values: a – the first set of specimens, b – the second set of specimens, c – the third set of specimens

As shown in Fig. 2 the compressive strength and the dynamics of hardening are similar for all sets of concrete specimens. This may be accounted for many factors: the amount of cement used, the relationship between the matrix and the aggregate, the conditions of testing, etc. Anyway, the experiment showed that the suggested method of aggregates composition design could be used for precise modelling of aggregates composition, providing the data for further research.

## CONCLUSIONS

1. The proposed method allows us to more accurately assess the effect of grading of aggregates on various properties of concrete.
2. The method offered can be used to design concrete with special properties.

3. The results of the experimental investigation show that compressive strength and dynamics of hardening are similar for all sets of concrete specimens.
4. Presented method of aggregates composition design, allows to reject ordinary methods of concrete design where aggregates of concrete are designed as two components (coarse aggregate and fine aggregate) mix. As a result, it could be used for non-conditional and cheaper components for concrete aggregates compositions.

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