Mathematical Correlations between Properties of Brick / in a Dried State (Part 1)

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In the first part of this work the relationships between physical properties of brick body are evaluated. These relationships are expressed by mathematical functions – correlation functions. Their accuracy is evaluated not only by arbitrary value of total porosity, but also at zero porosity. These correlation relations can be utilized for determining systematic error of used method or used device. This article should serve also as an instruction in choice of the correct mathematical function, which expresses a relation between the pairs of physical properties. In conclusion of the first part the possibility of their exploitation for practical purposes at the prompt checking directly at the brickworks is exhibited.
Keywords: brick, physical properties, pore structure, mathematical functions, dried state.

1. INTRODUCTION

The author had three reasons for writing of the first part of article. In the first place it was a concern to continue in extending the findings, which already were published in works [1, 2]. There the effort was about all to broaden the understanding of the relationships between the individual pairs of physical properties of the brick body. Many attempts for expressing of the relationships between physical properties of the material are published in the technical literature, too. Most frequently we can see such relationships between the pairs of physical properties [3 – 8]. These are usually expressed by mathematical formula on the basis of the data obtained experimentally. For example strength-porosity relationship belongs to those more frequently published.

In general, there exists a fundamental inverse relationship between porosity and the strength of solids which, for simple homogeneous materials, can be described by the expression

$$S = S_0 \, . \, e^{-kp} \, , \tag{1}$$

where S is the strength of the material which has a given porosity p; S_0 is the intrinsic strength at zero porosity; and k is a constant. For many materials the ration S / S_0 plotted against porosity follows the same curve. Actually, the strength-porosity relationship is applicable to a very wide range of materials [9].

There is the well known Powers`s relation, too. Powers found the 28-day compressive strength f_c of three different mortar mixtures was related to the gel/space ratio, or the ratio between the solid hydration products in the system and the total space:

$$f_c = ax^3 \, , \tag{2}$$

where a is the intrinsic strength of the material at zero porosity p and x the solid/space ratio or the amount of solid fraction in the system, which is therefore equal to $(1 – p)$ [10].

Sometimes we can translate also some types of mathematical functions trough experimentally measured values by the graphic expression of a relationship between the pairs of physical properties, see Fig. 1. It is mainly, when the experimentally measured values are located in a small interval. This article should therefore serve also as an instruction in choice of the correct mathematical function, which expresses a relation between the pairs of physical properties.

It is generally known that decisive effect on the physical properties of the brick body has the pore structure. However its values as for example the pore volume and median pore radius or total porosity are not easy to determine exactly by the present-day measurement methods. We can only state that our measurement can always contain the systematic and stochastic measurement errors. And it is the second reason, which the author wants to describe.

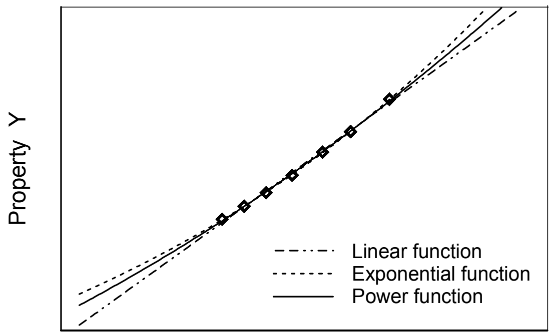

Fig. 1. Three development possibility of mathematical functions through given measured values

The last reason is the fact that nowdays we can not discover in case of many brickworks in lead time for example an error in the dosing of the raw material or in the firing process. Because there are constantly increased requirements on the quality of the finished product, the immediate control is one of the great postulates on the achieving this aim.

Unfortunately, many brickworks have not their laboratory equipped on such technical level, in which for

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example the compressive strength of the brick body, eventually of a finished product could be quickly determined. The perfect knowledge of the relationships among the individual pairs of the physical properties should be the assumption for the development such of test method, which could be used for the immediate quality control of the finished products. Author in this case proposes to use of the test methods, which are challenging neither from time, nor from financial aspect. Among these methods we could for example integrate the determination of water capillarity after a given contact time or of thermal conductivity by apparatus, where the determination value of the coefficient thermal conductivity is possible after a few minutes [1].

2. EXPERIMENTAL

Brick raw material from the locality Boleraz was used in the research. This raw material consists of eolian loess and neogene clay. Chemical composition is given in Table 1. Sawdust out of soft wood and paper sludges were used as the organic combustible admixture.

Table 1. Chemical composition of the used row materials

Chemical composition	Clay	Loess	Cellulose waste
SiO_2 [%]	59.94	65.09	16.27
Al_2O_3 [%]	11.95	12.20	10.45
Fe_2O_3 [%]	4.52	4.59	0.62
CaO [%]	7.48	4.66	18.69
MgO [%]	2.48	2.02	0.90
Na_2O %	0.84	1.05	0.10
K_2O [%]	2.14	2.06	0.53
CO_3 [%]	11.2	6.54	27.90
Loss on ignition [%]	8.88	6.31	51.57

The test specimens (dimension (100×50×20) mm) were manufactured from the mentioned above raw materials, where a mixing proportion of clay : loess = 70 : 30 vol.-%. (below as the fundamental raw material) and the mixing proportion of sawdust : paper sludges = = 70 : 30 vol.-%. (below as the organic combustible admixture). The organic combustible admixture to the fundamental raw material was added in doses of 1; 2; 3; 4; 5 and 6 wt-% (referred to the weight of the dried fundamental raw material). In all cases a constant plastic mass was kept (upsetting height 30 mm according to Pfefferkorn). For every admixture dose five test specimens were manufactured.

The green shaped products of the test specimens were at first pre-dried during 48 hours at a laboratory temperature (20 °C). During this time the values for Bigot curves were measured. The semi-dry shaped products were consequently dried in a laboratory-drying oven at a temperature of 105 °C to a constant weight. The test specimens were then fired in an electric laboratory furnace, where the firing temperature (900 °C) and the dwell time (1 hour) were constant, see Fig. 2.

The linear drying and firing shrinkage, the water absorption (boiling method) and the apparent porosity were determined according to standard STN 72 1565, Part 5 and Part 6. The water capillarity was measured after 1 hour. The test specimens were erected in petri dishes with water so, that the water level was 10 mm. The total porosity was measured by a pyknometer method and the compression strength on the test specimens with the dimension approximately (20 × 20 × 20) mm. An Isomet Model 104 (f. Applied Precision Ltd., Bratislava) was used to determine the thermal conductivity coefficient.

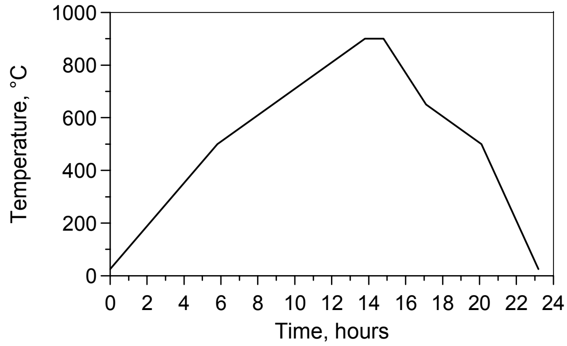

Fig. 2. Firing curve

The pore structure of the brick body was measured by a high-pressure mercury porosimeter (Model 1500 from Erba Science Co.).

2.1. Input data

The effect of the organic combustible admixture dose and the heating rate temperature on the selected physical properties of the clay-loess mass and brick body is given in Table 2. The most adequate mathematical function expressing the relationship among all the physical properties given in Table 2 and the admixture dose is a quadratic function, see Fig. 3.

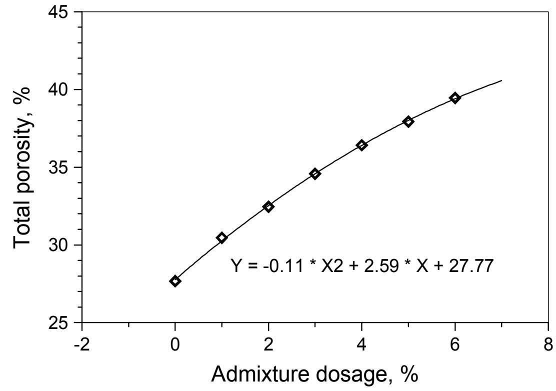

Fig. 3. Influence of the organic combustible admixture on the total porosity

3. RESEARCH METHOD

From the Table 2 the properties were selected for our other intention: pore volume and median pore radius, water capillarity, total and apparent porosity, bulk density, compression strength and thermal conductivity.

In the second step, we could think on the basis of known relation among apparent porosity, water absorption by boiling and bulk density:

$$P_z = NV \cdot \rho_v \tag{3}$$

Table 2. Physical properties of the test specimens with different dose of organic combustible admixture

Property	Dose of organic combustible admixture [wt.-%]						
	0	1	2	3	4	5	6
Working moisture content [%]	23.37	23.96	24.63	25.18	25.88	26.50	26.90
Linear drying shrinkage [%]	7.73	7.61	7.42	7.21	7.05	6.90	6.69
Drying sensitivity [-]	1.69	1.59	1.50	1.42	1.34	1.26	1.20
Linear firing shrinkage [%]	0.12	0.28	0.44	0.47	0.46	0.43	0.37
Weight loss by firing [%]	8.80	9.39	10.04	10.62	11.20	11.82	12.44
Water capillarity [mm/h]	17.5	22.5	25.0	29.0	32.5	35.0	37.5
Water absorption by boiling [%]	12.43	13.91	15.22	17.00	19.02	20.73	23.29
Apparent porosity [%]	23.49	25.40	27.09	29.41	32.09	34.23	37.61
Total porosity [%]	27.67	30.46	32.45	34.57	36.41	37.93	39.45
Bulk density [kg/m^3]	1890	1826	1780	1730	1687	1651	1615
Density [kg/m^3]	2613	2626	2635	2644	2653	2660	2667
Pore volume [mm^3/g]	130.2	146.5	159.5	174.3	187.0	198.8	210.7
Median pore radius [nm]	106.6	110.2	112.9	115.7	118.0	119.9	122.0
Compressive strength [MPa]	76.5	63.3	54.7	46.2	40.0	35.1	31.8
Thermal conduction coef. at 0% humidity [W/m.K]	0.693	0.638	0.603	0.564	0.534	0.511	0.489

where P_z is apparent porosity in wt.-%, *NV* is the water absorption by boiling in wt.-%, ρ_v is the bulk density of the brick body determined by hydrostatic method in kg/m^3 or g/cm^3 and *NV* is defined as 100 $(m_n - m_s)/m_s$, where m_n is the saturated weight of the specimen after 4 h, submersion in boiling water in kg or g, m_s is the the dry weight of the specimen in kg or g.

NV only with the apparent porosity and the bulk density will be next considered.

For the determination of mathematical relations between the pairs of physical properties the following methodical process was chosen. At first a specific control schema was formed (similar to that illustrated in Fig. 4), where the mentioned above selected properties were considered as well.

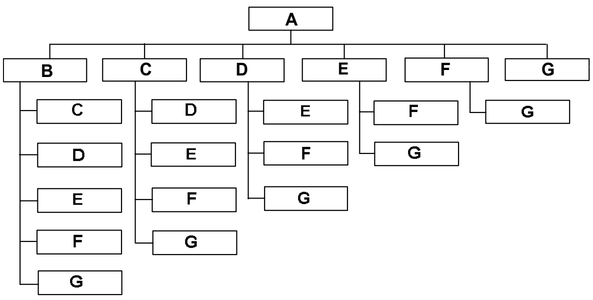

Fig. 4. Checking scheme for correlation functions between physical properties; **A** – total porosity, **B** – pore volume, **C** – median pore radius, **D** – water capillarity, **E** – bulk density, **F** – compressive strength, **G** – thermal conduction coefficient

The computer software (using the least-square method) was then applied for finding of the optimal mathematical functions between the individual pairs of physical properties according to the chosen control schema. Certain correlation functions for the majority of the pairs of physical properties were determined by the sequential control as is listed in the chapter 3.1. Afraid, it was not determined only for those pairs, in which occurs the apparent porosity. (It will be a subject of a further research.)

On the basis of this fact already with this property hereinafter wasn't considered and the new checking scheme was created as is illustrated in Fig. 4.

3.1. Testing of correlation functions

On the basis the control scheme shown in Fig. 4 the optimal correlation functions were determined between the pairs of physical properties, which mathematical notations are given in Table 3. The correctness of these correlation functions were in the next step verified in the following way. As an initial point was chosen an arbitrary value of the total porosity (**A** = 20 %) in area, in which were not the experimentally measured values. Though sequential substitution of the calculated values according to the scheme in Fig. 4 was achieved a specific control system that is shown in Fig. 5. By comparing of the values calculated for the every physical property we can state that correlation functions given in Table 3 are correct. The checking of given correlation functions was carried out also for **A** = 0 %. This checking only certified the correctness of correlation functions also in case, when the volume pore **B** and water capillarity **D** has a minus sign.

3.2. Systematic error of measurement

As it was already mentioned above, many applied methods for the determination of physical properties of the brick body can be characterized through a systematic measurement error. At the same time it is well known that this error can be either plus or minus sign. Among test methods for the determination of properties of the brick

Table 3. Correlation functions between the pairs of physical properties before modification of experimentally determined values

Designation correlation	Correlation function	Designation correlation	Correlation function
AB	$Y = 6.85339 \cdot X - 61.5069$	**CD**	$Y = 1.30623 \cdot X - 121.844$
AC	$Y = 1.30387 \cdot X + 70.5363$	**CE**	$Y = \exp(-0.0102463 \cdot X) \cdot 5647.47$
AD	$Y = 1.70391 \cdot X - 29.7331$	**CF**	$Y = \exp(-0.0581954 \cdot X) \cdot 38403.7$
AE	$Y = \exp(-0.0133629 \cdot X) \cdot 2741.69$	**CG**	$Y = \exp(-0.0227873 \cdot X) \cdot 7.87039$
AF	$Y = \exp(-0.0759055 \cdot X) \cdot 633.917$	**DE**	$Y = \exp(-0.00783306 \cdot X) \cdot 2170.87$
AG	$Y = \exp(-0.0297159 \cdot X) \cdot 1.57764$	**DF**	$Y = \exp(-0.0445009 \cdot X) \cdot 168.348$
BC	$Y = 0.189745 \cdot X + 82.3255$	**DG**	$Y = \exp(-0.0174162 \cdot X) \cdot 0.93867$
BD	$Y = 0.248149 \cdot X - 14.3593$	**EF**	$Y = \mathrm{pow}(X,5.6792) \cdot 1.89059E-017$
BE	$Y = \exp(-0.0019477 \cdot X) \cdot 2430.95$	**EG**	$Y = \mathrm{pow}(X,2.22192) \cdot 3.61946E-008$
BF	$Y = \exp(-0.0110601 \cdot X) \cdot 319.901$	**FG**	$Y = \mathrm{pow}(X,0.391103) \cdot 0.126384$
BG	$Y = \exp(-0.00432495 \cdot X) \cdot 1.20604$		

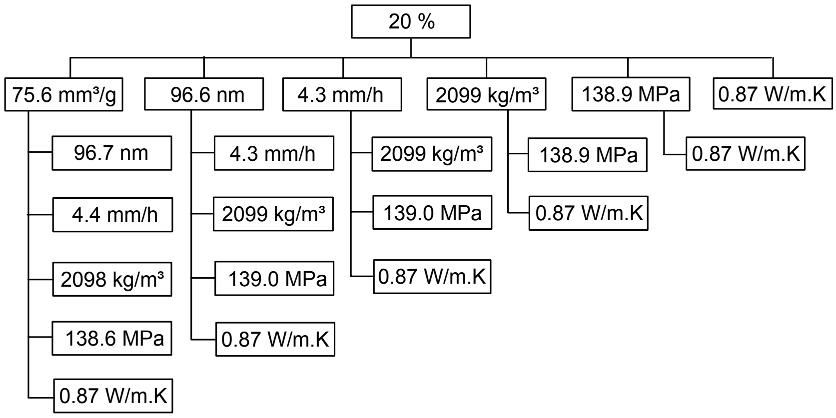

Fig. 5. Checking scheme for the correlation functions between physical properties at total porosity **A** = 20 %

body, at which it is considered with the systematic measurement error we can include for example:

- the determination of the pore volume and the median pore radius using a high pressure mercury porosimetry,
- the determination of the apparent porosity and the bulk density using a hydrostatic method,
- the determination of the density using the pycnometer method,
- the determination of the water absorption by boiling.

There are also the properties, where a value can be calculated on the basis of other two properties. If the values of these properties were obtained using the mentioned above test methods, then it's only natural that systematic measurement errors are transferred also into the new calculated value. For example, if the calculation of the total porosity is according to formula:

$$P_c = 1 - \frac{\rho_v}{\rho}, \qquad (4)$$

where P_c is total porosity in wt.-%, ρ_v is the bulk density in kg/m^3 or g/cm^3, ρ is the density in kg/m^3 or g/cm^3, are two properties in certain proportion, then we take into account three alternatives:

- the systematic measurement error increase,
- the systematic measurement error decrease,
- the systematic measurement error is eliminated.

It's only natural that in all previous calculations, which values are listed in Fig. 6, were these calculated with the specific systematic measurement errors.

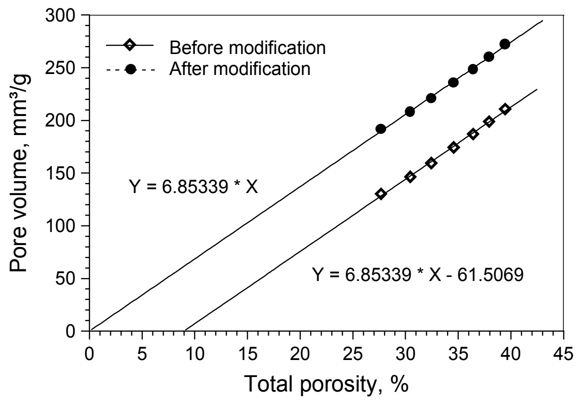

Fig. 6. Correlation functions between the total porosity **A** and pore volume **B** before and after modification of the experimentally determined values

The analyzing graphs of correlation functions between total porosity **A** and pore volume **B** or between total porosity **A** and median pore radius **C** demonstrate it as well, see Figs 6 and 7. Considering that the systematic measurement error is caused only by high pressure mercury porosimeter, then linear relations between total porosity **A** and pore volume **B**, eventually between total porosity **A** and median pore radius **C** should pass through the origin of coordinate system. On the basis this presumption, the authentic linear functions of the form:

Table 4. Values of the pore volume and median pore radius before and after modification of the experimentally determined values

Property	Dose of the organic combustible admixture [wt.-%]						
	0	1	2	3	4	5	6
Pore volume **B** [mm^3/g]	130.2	146.5	159.5	174.3	187.0	198.8	210.7
Pore volume **B** after modification [mm^3/g]	191.7	208	221	235.8	248.5	260.3	272.2
Median pore radius **C** [nm]	106.6	110.2	112.9	115.7	118.0	119.9	122.0
Median pore radius **C** after modification [nm]	36.1	39.7	42.4	45.2	47.5	49.4	51.5

$$Y = aX - b \tag{5}$$

or

$$Y = aX + b \tag{6}$$

should be modified to the form

$$Y = aX\,. \tag{7}$$

After this modification were values of the pore volume **B** and of the median pore radius **C** re-calculated and their new values are given in Table 4, see Figs 6 and 7.

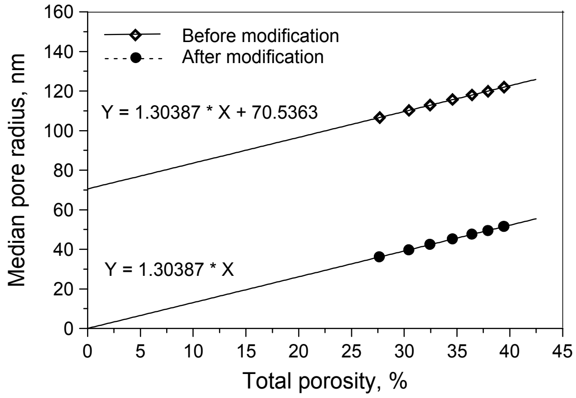

Fig. 7. Correlation functions between the total porosity **A** and median pore radius **C** before and after modification of the experimentally determined values

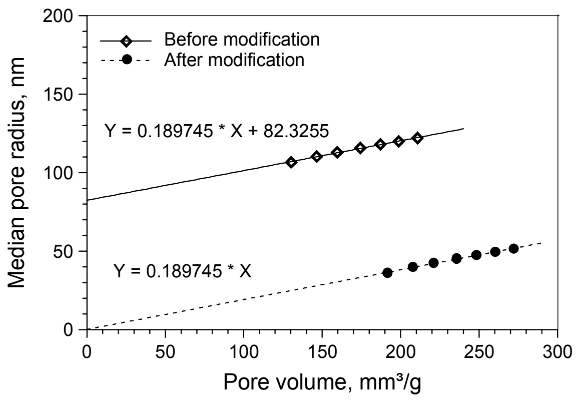

Fig. 8. Correlation functions between the pore volume **B** and median pore radius **C** before and after modification of the experimentally determined values

In another step was this whole procedure of control was repeated only with the modified values of the pore volume **B** and of the median pore radius **C** according to the checking scheme in Fig. 4. After these steps the modified correlation functions were obtained, which are given in Table 5 and the part of them is shown also in Figs 8 to 12.

Table 5. Correlation functions between the pairs of physical properties after modification of the experimentally determined values

Designation correlation	Correlation function
AB	$Y = 6.85339 \cdot X$
AC	$Y = 1.30387 \cdot X$
AD	$Y = 1.70391 \cdot X - 29.7331$
AE	$Y = \exp(-0.0133629 \cdot X) \cdot 2741.69$
AF	$Y = \exp(-0.0759055 \cdot X) \cdot 633.917$
AG	$Y = \exp(-0.0297159 \cdot X) \cdot 1.57764$
BC	$Y = 0.189745 \cdot X$
BD	$Y = 0.248149 \cdot X - 29.6205$
BE	$Y = \exp(-0.0019477 \cdot X) \cdot 2740.3$
BF	$Y = \exp(-0.0110601 \cdot X) \cdot 631.569$
BG	$Y = \exp(-0.00432495 \cdot X) \cdot 1.57354$
CD	$Y = 1.30623 \cdot X - 29.7548$
CE	$Y = \exp(-0.0102463 \cdot X) \cdot 2742.43$
CF	$Y = \exp(-0.0581955 \cdot X) \cdot 634.686$
CG	$Y = \exp(-0.0227873 \cdot X) \cdot 1.57871$
DE	$Y = \exp(-0.00783306 \cdot X) \cdot 2170.87$
DF	$Y = \exp(-0.0445009 \cdot X) \cdot 168.348$
DG	$Y = \exp(-0.0174162 \cdot X) \cdot 0.93867$
EF	$Y = \mathrm{pow}(X,5.6792) \cdot 1.89059E-017$
EG	$Y = \mathrm{pow}(X,2.22192) \cdot 3.61946E-008$
FG	$Y = \mathrm{pow}(X,0.391103) \cdot 0.126384$

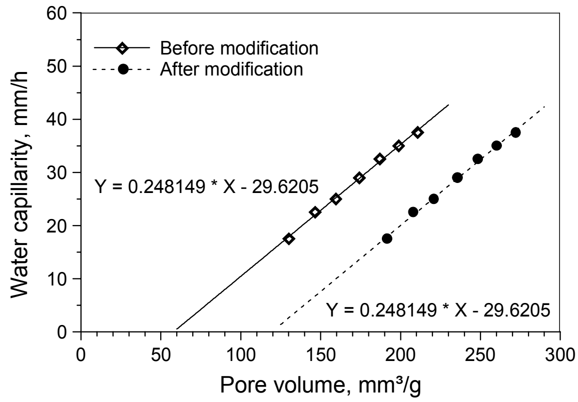

Fig. 9. Correlation functions between the pore volume **B** and water capillarity **D** before and after modification of the experimentally determined values

Table 6. Bulk density **E**, compressive strength **F** and thermal conductivity **G** determined at **A** = 0 %, **B** = 0 mm^3/g, **C** = 0 nm and 0 % humidity

Property	Designation correlation	Total porosity **A** = 0 %	Pore volume **B** = 0 mm^3/g	Median pore radius **C** = 0 nm
Bulk density **E** [kg/m^3]	**AE, BE, CE**	2742	2740	2742
Compressive strength **F** [MPa]	**AF, BF, CF**	633.9	631.6	634.7
Thermal conduction coefficient **G** [W/m.K]	**AG, BG, CG**	1.58	1.57	1.58

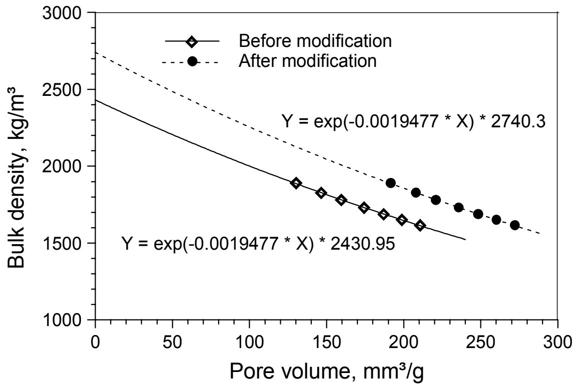

Fig. 10. Correlation functions between the pore volume **B** a bulk density **E** before and after modification of the experimentally determined values

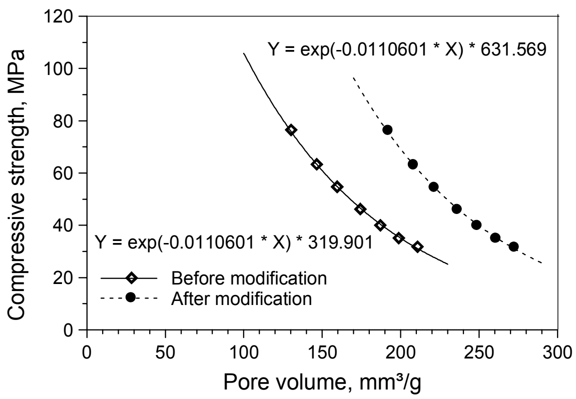

Fig. 11. Correlation functions between the pore volume **B** and compressive strength **F** before and after modification of the experimentally determined values

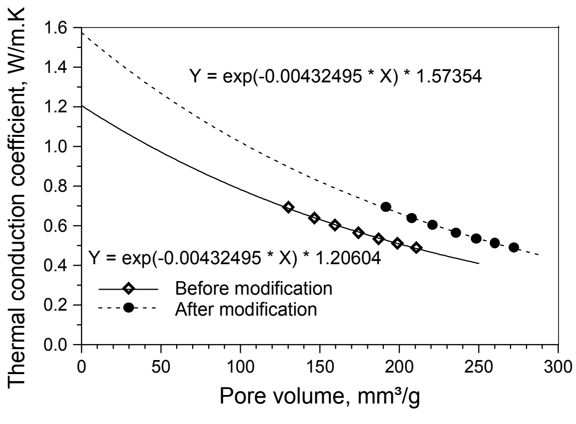

Fig. 12. Correlation functions between the pore volume **B** and thermal conductivity **G** before and after modification of the experimentally determined values

3.3. Testing of correlation functions in the boundary conditions

The control of relationships was carried out in the last step so that they were chosen: the total porosity **A** = 0 %, the pore volume **B** = 0 mm^3/g and the median pore radius **C** = 0 nm. Under these conditions the new values for the bulk density **E**, the compressive strength **F** and the thermal conductivity **G** were calculated, see Table 6. In those case the values of the bulk density **E** at the zero total porosity, at the zero volume pore and zero median pore radius should be actually to represent in all three cases the values of the brick body density. By comparing of the density values in Table 2 we can see that the recalculated values are much higher than the maximum value measured by pycnometer method. (The increase of density values is caused by the local growing of temperature caused by the increasing proportion of the present organic combustible material in the brick body.)

On the basis of the mutual comparison of the density values in Tables 2 and 6 we can state then that the density values measured by pycnometer method are affected by the systematic measurement error because they do not correspond to the actual values. From Table 6 it is also seen that not only the values of bulk density (density) **E**, but also the values of compressive strength **F** and thermal conductivity **G** obtained by "a various way" are almost identical. These results verify only a fact that even in “boundary conditions”, i.e. at **A** = 0 %, **B** = 0 mm^3/g and **C** = 0 nm used types of mathematical functions are very reliable.

By a control mechanism mentioned above was confirmed only a fact that values before modification of the pore volume, median pore radius and density, given in Tables 2 and 4 are values with the constant systematic measurement errors.

CONCLUSIONS

On the basis of the results achieved in this part of the article, in which relationships between the pairs of physical properties of the brick body were observed, we can state following conclusions:

1. The knowledges listed in [1, 2] were verified using other input raw materials and at the same time they were extended by the physical property of the water capillarity.
2. The relationship between the admixture dose and physical property of the brick body is possible to describe best in the most cases using a quadratic

function. Only in the case the linear firing shrinkage burning it wasn't possible.

3. The relationships between the individual pairs of physical properties can be described by three types of mathematical functions: linear, exponential and power, see Table 5. Here is however necessary to accent that for every pair of physical properties is possible to define only one type of mathematical function.
4. It was also verified that in the change of the brick clay, the organic combustible additive, the firing temperature, the types of mathematical functions always remain, only their coefficients change.
5. The types of mathematical functions listed in Table 7 are valid also at application of physical properties, which were determined with the systematic measurement errors.
6. The types of mathematical functions listed in Table 7 are valid only for the brick body without the black or reduction core, this means, the cross section of the brick body must be consistently homogeneous.

Table 7. Ground types of mathematical functions between the pairs of physical properties of the brick body

	A	B	C	D	E	F	G
A		LI	LI	LI	EX	EX	EP
B	LI		LI	LI	EX	EX	EP
C	LI	LI		LI	EX	EX	EP
D	LI	LI	LI		EX	EX	EP
E	EX	EX	EX	EX		PO	PO
F	EX	EX	EX	EX	PO		PO
G	EX	EX	EX	EX	PO	PO	

A – Total porosity
B – Pore volume
C – Median pore radius
D – Water capillarity
E – Bulk density
F – Compressive strength
G –Thermal conduction coefficient

LI – Linear function
EX – Exponential function
PO – Power function

In the determination of the values of total porosity by pycnometer method and the values of the pore volume or median pore radius by the high pressure mercury porosimeter it is necessary to consider that these values can be inaccurate, because they are usually loaded by systematic measurement errors. Using mentioned above the control methodology and the correlation functions between the pairs of physical properties it is possible to determine the very accurate values of pore volume and median pore radius as well as density of the brick body.

The cognition of correlation functions between the pairs of physical properties of the brick body would be possible to exploit also for the fast checkout right in a plant. By that control in a plant it could be followed regularly for example the compressive strength of the brick body. In this way it would be guaranteed the operative control of the input raw materials, which affect the quality of the brick product.

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