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The Method of Thermal Process Action Investigation in Materials Using the Descriptive Investigation of Effective Thermal Conductivity

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High-temperature conductive and convective processes are widely used in the different areas of technology. These processes directly influence the materials of the media, which must possess the specific heat insulation and erosional durability. Therefore it becomes urgent to create the procedures, which make it possible to experimentally establish a change in the erosional durability with respect to a change in the heat conductivity. The technique of erosion modeling process is offered by the model of heat conductivity. It allows to define an indirect quantitative measure of erosion on the basis of the factor of heat conductivity which is presented in the model equation. Thus process of erosion can be indirectly estimated during its development according to the in time varying factor of heat conductivity. In this work the model of thermal conductivity, which solves the problem of the indirect simulation of erosion is built and the method of solving the equation of model is given. The offered technique is actual for construction of systems of research and diagnostics of erosion process.

Keywords: erosion , thermal conductivity, high-temperature, model, identification, inverse problems.

INTRODUCTION

Studying the erosive destruction of the heated refractories and ceramic surfaces in high-speed streams of combustion products, is actual problem. Significant losses of weight of materials in such conditions lead to premature deteriorations to a design and structures of high-temperature devices and their unfitness to the further operation [1].

The refractory material heated up by the moving high-temperature gas stream up to appreciable temperature of sublimation, undergoes superficial and volumetric physical and chemical transformations [2]. Destruction eroding surfaces of materials develops of some balance of levels of external influence on the part of an accumulating gas stream and ability of a material to resist to this influence [3]. Superficial formations – "erosits", created by the moving environment, have characteristic components: a place tearing off a stream, a zone of connection of a stream, recirculation zone with craters, cavities, or poles. Such model of erosive destruction enables to assume distribution of indignation of pressure and mass streams of substance in a boundary layer on a rough surface. Mass speed of evaporation from a surface can be defined from expression Hertz-Knudsen [4]. During erosion, sublimation of eroding surfaces proceeds irregular. In different places of "erosits" there is a process of change of sublimation by condensation. It is caused diffusion and convection by ablation of sublimation products in an external stream from recirculation zones [3].

In erosive conditions of destruction, considering the specified model, pressure vapor in a cloud behind a deepening pressure sated pair will be essentially lower, similar to processes at sublimation in vacuum. This one has an extreme measures, or a limiting mode of possible process. In opposite case during destruction the other phenomena of equilibrium sublimation takes place which will be other extreme measure of possible ablation of weight of a material. Between these limiting modes of destruction, there is a transient site which passes the processes of kinetic up to equilibrium state depending on a "erosit" place in degrees of development of these formations. This process is inconceivable without considering a complex mass streams of substance with various irregularities creating the complex mechanism transfer of mass and promoting erosive destruction of a surface of a material [5].

It is necessary to note, that at the given investigation phase the assumption that the product of material sublimation – the particles which are taking place above eroding surface, are the components of products of combustion and behave as very fine particles with relaxation time. They have low mobility and achieve a speed equal to pulsation speed of gas. In this case the analogue of Schmidt number of a mixed gases flow with firm particles is equal to Schmidt's number for gases [3].

In a zone of connection of a stream there is a vertical inflow of energy to a wall [3]. After a stream connection the turbulence starts to grow in the field of a wall. This causes a maximal imbalance in generation and dissipation energy of turbulence which is observed in parietal layers of flow in a vicinity of a zone of connection. Just in this place most eroding active zone of "erosits" in which plastic flow of a material is observed [6]. The analysis of members of the equation of balance of kinetic energy of turbulence shows the important role of diffusion process, especially in a zone of connection of a stream. Mass of gas with the certain vertical speed are transferred to the surface layers, providing a force pulse to delayed layers.

On eroding surfaces of ceramics or refractories, with formation of roughnesses as "erosits", there is complex heat – mass exchange mechanisms. In same parts of "erosits" different mechanisms mass transfer and condensa-

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tion sublimate material takes place. Existence of streams and jets provides a heat and mass transfer and removal of sublimate and products of erosion from a surface. The change of pressure depending on the created obstacle promote process of destruction.

With all these phenomena intensive unilateral heat delivery, and energy of a high-temperature gas stream leaves in a ceramic or refractory wall are associated.

Recently researches of erosion of various materials and covering eroding surfaces are published in different scientific magazines. Modern research on destruction of eroding surfaces, definition of speeds of erosion and attempts to reduce them can be found in [7, 8]. The further research of erosion with the purpose of the prevention of possible materials surface destruction by erosion is still actual.

Heat exchange in the channel of a moving high-temperature gas stream basically will consist of several components: radiations, turbulent heat and mass exchange and heat conductivity. A quantitative estimation of erosion in progress is required. The application of turbulent heat exchange is related with the big difficulties of registration of this process. Proceeding from physical reasons we estimated, that the most informative way of supervision of influence of erosion on heat exchange process is a heat conductivity variation of a material during the erosion process.

Temperature gradient in a longitudinal direction of a gas stream in a kind of the smallness we ignore. The greatest heat exchange just also occurs to walls of a refractory material in a perpendicular direction to a stream in due to the greatest gradient in this direction. We postulate, that process of erosion influences heat conductivity of a normal direction to a wall of the channel. It can be proved by the following physical reasons. During erosion there is a formation of craters, cavities and poles on border between a high-temperature gas stream and a material surface. In these areas due to turbulence effects the temperature increases and as a consequence the thermal stream directed in a normal direction also raises. It can be interpreted as the heat conductivity factor increment which is presented in mathematical model. The increase in temperature influences on recrystallization and growth of grains in a frontier layer [3], also influences change of factor of heat conductivity.

We offer a technique of research of ceramics and refractories erosion on the basis of heat conductivity model in a material. The given technique allows to estimate quantitative characteristics of erosion from the moment of its origin and in development in time. We believe, that the offered technique will promote studying erosion resistance of materials and to creation of systems of erosion diagnosis.

STATEMENT OF A PROBLEM

Let some experimental installation making a high-temperature gas stream in the channel from a fire-resistant material which erosive stability is subject to definition during experiment is set. During experiment the temperature field is measured in a refractory material. Let the model of the phenomenon of heat conductivity in a researched refractory material is set.

The problem will consist in an estimation of factor of heat conductivity in time which process of erosion influences. The estimation of factor of heat conductivity in time is carried out on the basis of the measuring data and model of heat conductivity by the solution of identification problem of heat conductivity process linked with process of erosion. Thus we carry out indirect identification of process of erosion.

This problem it is possible to interpret just as coefficient-type inverse problem of heat conductivity. The task in view will consist of two important auxiliary problems:

1. Construction of model of the heat conductivity linked with a process of erosion.
2. Identification of process of the heat conductivity linked with a process of erosion.

PHYSICAL AND MATHEMATICAL MODEL OF PROCESS OF EROSION

Proceeding from statement of a problem, it is offered to model process of erosion on the basis of model of heat conductivity. For this purpose it is postulated the following statements:

1. During erosion there is a destruction of a surface of a material in a gas stream that influences the basically change of factor of heat conductivity in a researched material in radial direction.
2. Change of radial factor of heat conductivity can serve as a quantitative measure of an estimation of erosion as the phenomena during its development.

Hence for modeling process of erosion we use model of heat conductivity, and for a quantitative estimation of erosion it is necessary to receive an estimation of factor of heat conductivity in a radial direction. This problem is a problem of identification, or factor inverse problem of heat conductivity [9]. The modern solution of these inverse problems of heat conductivity can be found in [10 – 15].

According to factor of heat conductivity using a method descriptive regularization authors have published some results in [16 – 19]. The offered method of descriptive regularization allows to raise considerably the accuracy of an estimation of factor of heat conductivity by implementation of descriptive attributes which are set on the basis of known physical assumptions in the identification procedure.

Let us examine the section of the channel with the ceramic walls, inside which (rectangle *ABKP*) (Fig. 1) flows the high-temperature high-speed flow of combustion products. The flow of gas goes in the direction indicated and heats up sides *AB*, *KP*. Heat flux in the wall from the eroding surface it goes in the normal direction to the side of the external surface *CO*.

We will approximate three-dimensional heat propagation by the two-dimensional equation, which describes heat propagation in the rectangular region *ABCO*. This is caused by the fact that in similar type experiments, a quantity of sensors is limited in the space. Furthermore a superfluous quantity of sensors strongly distorts temperature fields and heat fluxes. Therefore it is necessary to find a reasonable compromise between a quantity of sensors and the accuracy of measurements. We consider that this reasonable compromise it is possible to find by the arrangement of sensors in the specific normal plane. In the case in question this is rectangle *ABCO*.

Fig. 1 shows the arrangement of sensors. Sensors Θ_{b1}, Θ_{b2}, ... , Θ_{b12} serve for registering the boundary conditions. The sensors Θ_{b1}, Θ_{b2}, ... , Θ_{b4} are located not directly on boundary of *AB*, but at a distance $d = A - A'$ from the boundary on line $A'B'$.

This by the fact that sensors on boundary of *AB* caused undergo additional actions with the erosion. They are the source of turbulent disturbances because of the being appeared heterogeneities. Furthermore they simply spoil with the direct effect, i.e. their lifetime is shortened. However, this is inadmissible, since they must be operational throughout prolonged experiment.

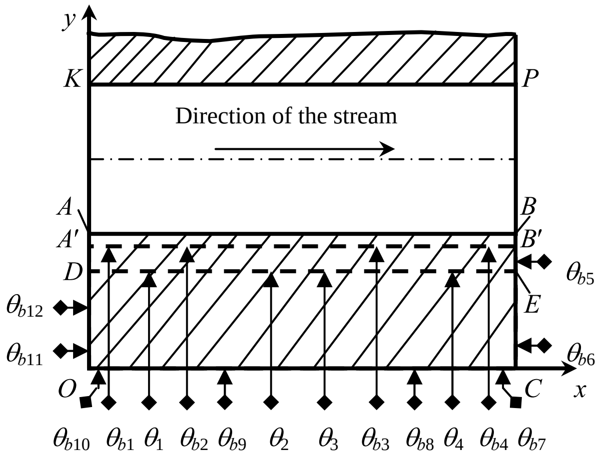

Fig. 1. Section of channel with the ceramic walls inside which flows the high-temperature high-speed flow of combustion products

With the arrangement of sensors on line $A'B'$, we will assume also that the geometry of boundary conditions will not change because of the erosion of surface of *AB*.

We will consider that the flow of high-temperature gas is stationary, if the process of erosion does not occur. Consequently in this case the process of thermal conductivity in region $A'B'CO$ also must be stationary. Upon consideration of the process of erosion, we deal concerning the nonstationary process of thermal conductivity, as a result of which changes the coefficient of thermal conductivity in the material.

The two-dimensional equation of thermal conductivity can be represented in the form

$$\rho c \frac{\partial T(x,y,t)}{\partial t} = \lambda_x \frac{\partial^2 T(x,y,t)}{\partial x^2} + \frac{\partial}{\partial y}\left(\lambda_y(T,t) \frac{\partial T(x,y,t)}{\partial y} \right), \quad (1)$$

where ρ is the material density, c is the thermal capacity, $\lambda_x = \text{const}$ is the the coefficient of thermal conductivity in direction x, $\lambda_y(T, t)$ is the coefficient of thermal conductivity in direction y, $T(x, y, t)$ is the temperature, dependent on coordinates x, y and time t.

To the equation in question the following boundary and initial conditions are added:

$$T(x,y,t)\big|_{x,y\in\Omega} = T_\Omega(t); \quad (2)$$

$$T(x,y,t)\big|_{t=0} = T_0(x,y), \quad (3)$$

where Ω is the bounding area given by rectangle $A'B'CO$.

Since we assume that the flow of gas stationary, temperature along the straight line $y = y_0 \le A'$ changes this change can be disregarded. Therefore composing thermal conductivities λ_x does not depend on the coordinate x and time t.

With a significant temperature difference on boundaries of $A'B'$ and *OC*, heat flux will flow from *AB* to *OC* and its intensity will depend on thermal conductivity λ_y, which in this case depends on temperature T and time t, assuming that the process of erosion, occurs.

The influence of thermal conductivity λ_x must be considerably below λ_y, since the gradient in direction x appears considerably smaller than in coordinate y.

In this model we postulate that in the course of time occurs the process of erosion and as consequence changes the coefficient of thermal conductivity λ_y, which draws a change in temperature field $T(x, y, t)$.

In the model the source functions and drains are not provided, since there are no sources, but the heat of sink through the measuring sensors are negligible. For applying the procedure of the identification of coefficients of λ_x and λ_y, it is necessary to produce repeated solution of equation (1), with add conditions (2), (3).

Let us examine a question of the approximation of boundary conditions according to measured data. Boundary conditions are approximated according to measured temperatures Θ_{b1}, Θ_{b2}, ... , Θ_{b12}. Let us first examine the approximation of values Θ_{b1}, Θ_{b2}, ... , Θ_{b4} on boundary of A′B′. For this are used basic splines of first order $\beta_0(x)$, $\beta_1(x)$, ... , $\beta_4(x)$ expressed in the form

$$\varphi_{AB}(x) = \sum_{i=0}^{4} b_i \beta_i(x) \quad (4)$$

minimizing the following quadratic criterion of the quality

$$Q_{AB}(b) = \sum_{k=1}^{4} \left(\varphi_{AB}(x_k) - \Theta_k\right)^2 = \sum_{k=1}^{4} \left(\sum_{i=0}^{4} b_i \beta_i(x_k) - \Theta_k \right)^2, \quad (5)$$

where $b = (b_0, b_1, ... , b_6)$ is the desired vector of the parameters, on which φ_{AB} and thus Q_{AB} depends.

However, the unknown function $\varphi_{AB}(x)$ must smoothly be connected with other functions $\varphi_{OA}(x)$ and $\varphi_{BC}(x)$ on end-points A' and B'. Analogous condition must satisfy function on boundary of *OC*.

Therefore it is the best to produce approximation for all measuring points $\Theta_{b1}, \Theta_{b2}, ... , \Theta_{b12}$, mentally straightening rectangle A′B′CO into the straight line in direction x. Then instead of $\varphi_{AB}(x)$, we obtain $\varphi(x)$, where $\varphi(x_1) = \Theta_{b1}$, $\varphi(x_2) = \Theta_{b2}$, ... , $\varphi(x_{12}) = \Theta_{b12}$, and the straightened rectangle is torn up at point A'. Initial point is A' of which it corresponds to $x = 0$, and final we will designate A'', to which $x = a$ corresponds. Then $\varphi(x)$ is determined in interval of $x = [0, a]$ and it is expressed by the equation:

$$\varphi(x,t)=\sum_{i=0}^{n} b_i(t)\beta_i(x), \qquad n=13 \tag{6}$$

with the boundary condition

$\varphi(x=0,t)=\varphi(x=a,t)$.

It is assumed that $\varphi(x,t)$ must approximate boundary condition (2). Coefficients $\{b_i(t)\}$ are selected in the form of the polynomials of the third power in the form:

$b_i(t)=\alpha_{0i}+\alpha_{1t}\cdot t+\alpha_{2i}\cdot t^2+\alpha_{3i}\cdot t^3$.

Then function depends on the desired vector of the parameters

$\alpha=(\alpha_{00},\alpha_{01},...,\alpha_{03},\alpha_{10},...,\alpha_{13},...,\alpha_{n0},...,\alpha_{n3})$.

We will use the minimization criterion of quality in the form:

$$Q(\alpha)=\sum_{k=1}^{K}\sum_{i=1}^{n}\left[\varphi(x_i,t_k)-\Theta_{bi}(t_k)\right]^2, \tag{7}$$

where $\Theta_{bi}(t_k)$ is the indication of sensor Θ_{bi} at the moment of time t_k, K is the number of measurements on the time, which is determined by the duration of experiment. The estimation of that desired the vector of the parameters α', is found by the minimization of criterion Q, i.e.:

$$\alpha'=\min_{\alpha} Q(\alpha). \tag{8}$$

This is reached by usual methods [20], therefore we not will discuss this question and will consider that the function $\varphi(x,t)$ is obtained and it is approximated by boundary conditions (2). It is further necessary to approximate initial condition (3) on measurements Θ_{b1}, Θ_{b2}, ... , Θ_{b12} and Θ_1, Θ_2, Θ_3, Θ_4. Approximation we will produce with two-dimensional splines of the first order [21]:

$B_i(x,y)=a_{0i}+a_{1i}\cdot x+a_{2i}\cdot y$,

where $x,y\in\Omega_{\Delta i}$ it is triangular region on rectangle $A'B'CO$. The separation of rectangle $A'B'CO$ into the triangles $\{\Omega_{\Delta i}\}$ is shown in Fig. 2. As it is evident partition into the triangles occurs in such a way that measured points Θ_1, Θ_2, Θ_3, Θ_4 correspond to the apexes of triangle. As a result we obtain 20 regions $\Omega_{\Delta i}$. Then the unknown function $\Phi(x,y)$, which interpolates initial condition (2) it is possible to express in the form:

$$\Phi(x,y)=\sum_{i=1}^{N} c_i B_i(x,y). \tag{9}$$

With the partition of rectangle $A'B'CO$ we obtain 18 points of the apexes of rectangles. To any point of the partition (x_k,y_l) of rectangle is placed in the correspondence function $B_i(x,y)$, which is equal to one at point (x_k,y_l), and at remaining points it is equal to zero, where $k=0,1,...,5$; $l=0,1,2$.

Thus we do obtain $N=6\cdot 3=18$ functions $B_i(x,y)$, which we will express in the analytical form. For an example let us take triangular region $\Omega_{\Delta k}$ and write down formula for function $B_i(x,y)$. Let us assume that $\Omega_{\Delta k}$ is assigned by three points (P_i, P_n, P_m), and its value at point $P_i=(x_i,y_i)$ it is equal to one. The numeration of points can be produced from left to right and from bottom to top. Then

$$B_i=(x,y)=\left(1-\frac{y_i-y_m}{y_n-y_m}-\frac{x_i-x_n}{x_m-x_n}\right)^{-1}\times \\ \times\left(1-\frac{y-y_n}{y_n-y_m}-\frac{x-x_n}{x_m-x_n}\right). \tag{10}$$

For determining the coefficients, it is possible to use the method of least squares [20]. The quadratic criterion of the quality is determined for this

$$Q_c=\left[\sum_{i=1}^{N} c_i B_i(x_k,y_l)-T_{k,l}\right]^2, \tag{11}$$

where – $k=0,2,...,5$; $l=0,1,2$, $T_{k,l}$ are the measured temperatures at the initial moment of time $t=0$ at points x_k,y_l. Estimation c' of the desired vector of the parameters $c=(c_1,c_2,...,c_N)$ is found from the condition:

$$c'=\min_{c} Q_c, \tag{12}$$

which can be found by solving the system of linear equations, which corresponds to the second-order conditions of the minimum of criterion Q_c.

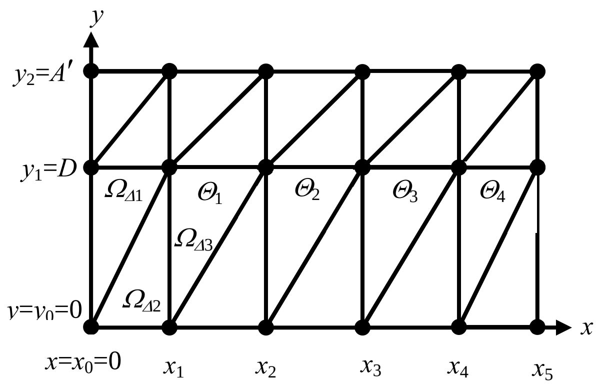

Fig. 2. Triangulation of region and computational grid for solving the equation of thermal conductivity

By having functions $\varphi(x,t)$ and $\Phi(x,y)$ as the boundary and initial conditions, determined during the experiment, it is possible to approach the solution of equation (1).

This equation can be solved by grid [22] or variational methods [21]. We will further assume that we in the state it to solve having the assigned coefficients of thermal conductivity λ_x and $\lambda_y(T,t)$. Having a solution of $T(x,y,t)$ on a certain set of three-dimensional points of the x_i, y_i and at the moments of time t_k, it is possible to approach the process of the identification of the coefficients of thermal conductivity λ_x and $\lambda_y(T,t)$.

For solution (1) and identification it is necessary to produce parametrization $\lambda_y(T,t)$ via its suitable approximation. We will express this coefficient in the form

$$\lambda_y(T,t)=s_0(T)+s_1(T)t+s_2(T)t^2+s_3(T)t^3, \tag{13}$$

where functions, are expressed in the form:

$$s_i(T)=\tau_{0i}+\tau_{1i}\cdot T+\tau_{2i}\cdot T^2. \tag{14}$$

Consequently $\lambda_y(T,t)$ is assigned by a vector $s=(s_0,s_1,s_2,s_3)$, whose component are determined by parameters $\tau_{0i}, \tau_{1i}, \tau_{2i}$, $i = 0,\dots,3$. A common vector we let us designate $\lambda = (r, s)$.

SOLUTION OF THE EQUATION OF MODEL

Equation (1) with additional conditions (2), (3) is quasi-linear equation; therefore its solution is achieved by an iteration technique, after assigning the initial condition $\lambda_{y,0}(T_0, t)$. After obtaining solution $T_1 = T(x, y, t; \lambda_{y,0})$, using (13), we determine the new value of $\lambda_{y,1}(T_1, t)$, which is original value for the second iteration with calculation $T_2 = T(x, y, t; \lambda_{y,1})$. Process continues as long as it is not reached one of two or both conditions simultaneously

$$\left\| \lambda_{y,i+1}(T_{i+1}, t) - \lambda_{y,i}(T_i, t) \right\| \le \varepsilon_\lambda > 0 \,, \tag{15}$$

$$\left\| T_{i+1} - T_i \right\| \le \varepsilon_T > 0 \,, \tag{16}$$

where $\|\cdot\|$ is the standard of functions λ, T specific in spatial domain Ω = A′B′CO, and in the interval of time $\tau = [0, t_k]$ which can be selected in the form:

$$\left\| f(x, y, t) \right\| = \left[\int_\Omega \int_\tau f(x, y, t)^2 \, dxdydt \right]^{1/2} . \tag{17}$$

For this with the calculation of the standard of function $\lambda(T, t)$ it is transformed into function $\lambda(T(x, y, t), t) = \lambda'(x, y, t)$.

In general case it is difficult to obtain theoretical conditions of the convergence of iterative process. However, practically it is possible to consider that if with the given sufficiently low values of ε_λ and ε_T, inequality (15) and (16) they are satisfied after a certain number of iterations, then iterative process converges. Coefficients ε_λ and ε_T are called the criteria of the stop of the iterative process of solving the equation of model.

Let us build the diagram of the solution of equation (1) for one iteration. For this we will use the method of Galerkin [21]. Let us create new uniform discrete grid in spatial domain Ω, assigned by coordinates $\{x_i\}, \{y_j\}, i = 0, 1, \dots, N, j = 0, 1, \dots, M$. As a result we obtain rectangular grid from $N \cdot M$ squares, assigned by the coordinates $\{x_i\}$, $\{y_j\}$. Each square is divided into two triangles. A quantity of triangular subregions is equal $L = 2N \cdot M$.

Thus, analogous to (9), (10) it is possible to build L of the basic functions $B_i(x, y)$ on each triangular subregion.

We will search for the solution in the form:

$$T(x, y, t) = \sum_{i=1}^{L} c_i(t) B_i(x, y) \tag{18}$$

where the coefficients $c_i(t)$ be subject to determination.

Thus, the solution we search for in the function space of those determined on Ω, quadratically integrated first by the derivatives $H^1(\Omega)$. Consequently $\partial T / \partial x$ and $\partial T / \partial y$ are the quadratically integrated functions.

After substituting (18) in (1), we obtain:

$$\rho c \sum_{i=1}^{L} c_i'(t) B_i(x, y) - \lambda_x \sum c_i(t) \frac{\partial^2 B_i(x, y)}{\partial x^2} - \frac{\partial}{\partial y} \left(\lambda_y(T, t) \frac{\partial B_i(x, y)}{\partial y} \right) = 0 \,, \tag{19}$$

where $c_i'(t)$ – time derivative of coefficient $c_i(t)$.

Following the Galerkin method let us multiply (19) scalar for each of L of the basic functions $B_i(x, y)$, and the second and third members of equation let us integrate in parts, using boundary conditions. As a result we will obtain system from L of ordinary differential equations:

$$Ac'(t) + B(t)c(t) = 0 \,, \tag{20}$$

where matrix $A = \{a_{ij}\}$ is obtained as a result of scalar products of functions $B_i(x, y)$ with functions $B_j(x, y)$, and matrix $B(t) = \{b_{ij}\}$ is determined by scalar products $\lambda_x \frac{\partial B_i(x, y)}{\partial x}$ with $\frac{\partial B_j(x, y)}{\partial x}$ and $\lambda_y(T, t) \frac{\partial B_i(x, y)}{\partial y}$ with functions $\frac{\partial B_j(x, y)}{\partial x}$. Scalar products for any functions $f(x, y), g(x, y) \in H^1(\Omega)$ are determined in the form:

$$\left\langle f(x, y), g(x, y) \right\rangle = \int_\Omega f(x.y) g(x, y) dxdy \,. \tag{21}$$

Let us find initial conditions for (20) on the basis of the condition that function $T_0(x, y)$ from (3) as the desired solution $T(x, y, t)$, must be orthogonal projection on the space of the basic functions $\{B_i(x, y)\}$. Let $c_i(0) = c_i^0, i = 1, \dots, L$.

Then, on the basis of the condition of the orthogonality of the vector of the discrepancy of the space of the solutions, generated by basic functions, we obtain the following equation for determining the vector of the initial coefficients $\{c_i^0\}$:

$$Cc^0 = d \,, \tag{22}$$

where the matrix $C = \{c_{ij}\} = \int_\Omega B_i(x, y) B_j(x, y) dxdy$, $c^0 = (c_1^0, c_2^0, \dots, c_L^0)$ is the the vector of the unknown initial conditions, $d = (d_1, d_2, \dots, d_L)$,where $d_i = \int_\Omega T_0(x, y) B_i(x, y) dxdy$.

Having a matrix C and a vector of right side d, we find the desired vector of initial conditions in the form

$$c^0 = C^{-1} d \,. \tag{23}$$

For obtaining solution (20) with initial conditions (23), we convert it to the form:

$$C(t) = D(t)c(t) \,, \tag{24}$$

where $D(t) = -A^{-1} B(t)$.

Last equation can be solved by the method of Crank-Nicholson [21]. According to this, solution $c^{i+1} = (c_1^{i+1}, c_2^{i+1}, \dots, c_L^{i+1})$ in $(i + 1)$ – moment of time is determined from the equation:

$$\frac{1}{\tau}\left(c^{i+1} - c^i\right) + D^i \frac{1}{2}\left(c^{i+1} + c^i\right) = 0 \,, \tag{25}$$

where according to (23) is determined the initial condition c^0, $\tau = t_{i+1} - t_i$ it is the step of sampling.

Equation (4.11) permitted relative to c^{i+1}. Then

$$c^{i+1} = \left(E + \frac{1}{2}\tau D^i \right)^{-1} \left(E - \frac{1}{2}\tau D^i \right) c^i , \qquad (26)$$

where E – unit matrix.

The conditions of the convergence of grid scheme (26) can be found in [21]. Having the presented method of solving the model of the process of thermal conductivity, on basis the identification procedure, it is possible to simulate the process of erosion. A change of the coefficient of thermal conductivity in the time $\lambda_y(T,t)$ is the measure for erosion in this case. It is consequently necessary to find a change of this coefficient with time in the flow of physical experiment according to measured temperatures by sensors Θ_1, Θ_2, Θ_3, Θ_4. According to [9] this corresponds coefficient inverse problem, or the identification of thermal conductivity problem.

CONCLUSIONS

The procedure of the study of the erosion of ceramics and fireproof materials on the basis of the model of the phenomenon of thermal conductivity in the material is proposed. This procedure makes it possible to estimate the quantitative characteristics of erosion from the moment of its origin and in the process of development in the time. We assume that the proposed procedure will contribute to the study of the erosional durability of materials and to the creation of the systems of diagnostics of erosion.

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