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Optimal Sizing and Stacking Sequence of Composite Drive Shafts

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In this work an attempt has been made for design optimization of composite drive shafts for power transmission applications. The one-piece composite drive shaft is designed to replace conventional steel drive shaft of an automobile using E-glass / epoxy and high modulus (HM) carbon/epoxy composites. A formulation and solution technique using genetic algorithms (GAs) for design optimization of composite drive shafts is presented here. The purpose of using GA is to minimize the weight of shaft that is subjected to the constraints such as torque transmission, torsional buckling capacities and fundamental lateral natural frequency. The weight savings of the E-glass / epoxy and high modulus carbon/epoxy shaft were 48.36 % and 86.90 % of the steel shaft respectively.

Keywords: design optimization; drive shafts; automobile; composites; GAs; weight savings

1. INTRODUCTION

The advanced composite materials such as graphite, carbon, kevlar and glass with suitable resins are widely used because of their high specific strength (strength / density) and high specific modulus (modulus / density)[1]. Weeton et al. [2] described the application possibilities of composites in the field of automotive industry as elliptic springs, drive shafts, leaf springs etc., Beard more et.al [3, 4] highlighted the potential for composites in structural automotive applications from a structural point of view. Andrew Pollard [5] proposed the polymer matrix composites in driveline applications. Hurd [6] discussed in detail the torsional performance of drive shafts for vehicle driveline applications.

A GA proposed by Goldberg [7] based on natural genetics has been used in this work. In the previous study by the authors [9], GAs applied for the design optimization of steel leaf springs. Although design optimization of steel springs and composite leaf springs has been the subject for quite few investigators [9, 10], no paper has been reported (to the best of the knowledge of the authors) on composite drive shafts using the GA approach. However there is some literature using GAs [11 – 14]. GA is fairly new and is described in greater detail in the literature [7, 8].

Almost all automobiles (at least those which correspond to design with rear wheel drive and front engine installation) have transmission shafts shown in Fig. 1 [15]. The weight reduction of the drive shaft can have a certain role in the general weight reduction of the vehicle and is a highly desirable goal, if it can be achieved without increase in cost and decrease in quality and reliability. It is possible to reduce the weight of the drive shaft considerably by optimizing the design parameters by satisfying the all constraints

Conventional steel drive shafts [16] are usually manufactured in two pieces to increase the fundamental bending natural frequency because the bending natural frequency of a shaft is inversely proportional to the square of beam length and proportional to the square root of specific modulus. Therefore the steel drive shaft is made in two sections connected by a support structure, bearings and U-joints and hence over all weight of assembly will be more. Also, they have less specific modulus, specific strength and its corrosion resistance is less as compared with composite materials.

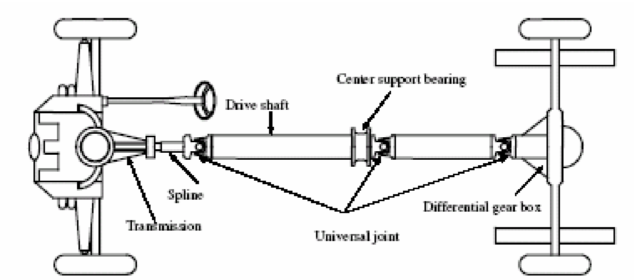

Fig. 1. Conventional two-piece drive shaft arrangement for rear wheel vehicle driving system

Advantages of composite drive shafts [17] includes: significant weight reduction, reduced bearing & journal wear, symmetric composite assures dynamic balance & increased operating speeds, electrically conductive or non-conductive, custom end-fitting configurations, corrosion resistant, reduced noise, vibration & harshness (NVH), long fatigue life.

In the present work an attempt is made to evaluate the suitability of composite material such as E-glass / epoxy and HM-carbon / epoxy for the purpose of automotive transmission applications. A one-piece composite drive shaft for rear wheel drive automobile was designed optimally by using GA for E-glass / epoxy and HM-carbon / epoxy composites with the objective of minimization of weight of the shaft which is subjected to the constraints such as torque transmission, torsional buckling strength capabilities and natural bending frequency.

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2. SPECIFICATION OF THE PROBLEM

The torque transmission capability of the drive shaft for passenger cars, small trucks, and vans should be larger than 3500 Nm (T_{max}) and fundamental natural bending frequency of the drive shaft should be higher than 6500 rpm (N_{max}) to avoid whirling vibration. The drive shaft outer diameter d_o should not exceed 100 mm due to space limitations. Here outer diameter of the shaft is taken as 90 mm. The drive shaft of transmission system was designed optimally to the specified design requirements [18].

3. DESIGN OF STEEL DRIVE SHAFT

Presently, steel (SM45C) is used for making automotive drive shafts. The material properties of the steel (SM45C) are given in Table 1 [19]. The steel drive shaft should satisfy three design specifications such as torque transmission capability, buckling torque capability and bending natural frequency.

Table 1. Mechanical properties of steel (SM45C)

Mechanical properties	Symbol	Steel
Young's modulus (GPa)	E	207.0
Shear modulus (GPa)	G	80.0
Poisson's ratio	ν	0.3
Density (Kg/m^3)	ρ	7600
Yield strength (MPa)	S_y	370
Shear strength (MPa)	S_s	370

3.1. Torque transmission capacity of the steel shaft

Torque transmission capacity T of a steel drive shaft is given by:

$$T = S_s \frac{\pi\left(d_o^4 - d_i^4\right)}{16 d_o} , \tag{1}$$

where S_s is the shear strength, d_o and d_i represent outside and inside diameter of the steel shaft.

3.2. Torsional Buckling Capacity of steel Shaft

If $\frac{1}{\sqrt{1-\nu^2}} \frac{L^2 t}{(2r)^3} > 5.5$, it is called as long shaft other wise it is called as short & medium shaft [20]. For a long shaft, the critical stress τ_{cr} is given by:

$$\tau_{cr} = \frac{E}{3\sqrt{2}(1-\nu^2)^{3/4}} (t/r)^{3/2} \tag{2}$$

where E, and ν represent steel properties. L, t and r are the length, thickness and mean radius of the shaft respectively. The relation between the torsional buckling capacity T_{cr} and critical stress is given by:

$$T_{cr} = \tau_{cr} 2\pi r^2 t \tag{3}$$

3.3. Lateral Vibration

The shaft is considered as simply supported beam undergoing transverse vibration or can be idealized as a pinned-pinned beam. Natural frequency f_{nt} is calculated using Timoshenko beam theory [21]. It considers both transverse shear deformation as well as rotary inertia effects. Natural frequency based on the Timoshenko beam theory is given by:

$$f_{nt} = K_s \frac{30\pi p^2}{L^2} \sqrt{\frac{Er^2}{2\rho}} , \tag{4}$$

where f_{nt} is the natural frequency and p is the first natural frequency. E and ρ are the material properties of the steel shaft, and K_s is given by:

$$\frac{1}{K_s^2} = 1 + \frac{p^2 \pi^2 r^2}{2L^2}\left[1 + \frac{f_s E}{G}\right], \tag{5}$$

where G is the rigidity modulus of the steel shafts and $f_s = 2$ for hollow circular cross-sections.

Critical speed:

$$N_{crt} = 60 f_{nt} . \tag{6}$$

4. DESIGN OF COMPOSITE DRIVE SHAFT

4.1.1. Selection of crosssection and materials

The following assumptions were made in our calculations:

- The shaft rotates at a constant speed about its longitudinal axis;
- The shaft has a uniform, circular cross section;
- The shaft is perfectly balanced, i.e., at every cross section, the mass center coincides with the geometric center;
- All damping and nonlinear effects are excluded;
- The stress-strain relationship for composite material is linear & elastic; hence, Hook's law is applicable for composite materials;
- Since lamina is thin and no out-of-plane loads are applied, it is considered as under the plane stress.

The drive shaft can be solid circular or hollow circular. Here hollow circular cross-section was chosen because the hollow circular shafts are stronger in per kg weight than solid circular and the stress distribution in case of solid shaft is zero at the center and maximum at the outer surface while in hollow shaft stress variation is smaller. In solid shafts the material close to the center are not fully utilized.

The E-glass / epoxy, high strength carbon / epoxy and high modulus carbon / epoxy materials are selected for composite drive shaft. Table 2 shows the properties of the E-glass / epoxy and high modulus carbon / epoxy materials used for composite drive shafts.

E_{11}, E_{22}, G_{12}, σ^T_1, σ^C_1, σ^T_2 and σ^C_2 represent lamina properties in longitudinal and transverse directions (Fig. 2) respectively. ν_{12}, τ_{12}, ρ and V_f are the Poisons ratio, shear stress and fiber volume fractions.

Table 2. Mechanical properties of E-glass / epoxy and HM carbon / epoxy

Property	E-glass / epoxy	HM carbon / epoxy
E_{11} (GPa)	50.0	190.0
E_{22} (GPa)	12.0	7.7
G_{12} (GPa)	5.6	4.2
ν_{12}	0.3	0.3
$\sigma^T{}_1 = \sigma^C{}_1$ (MPa)	800.0	870.0
$\sigma^T{}_2 = \sigma^C{}_2$ (MPa)	40.0	54.0
τ_{12} (MPa)	72.0	30.0
ρ (kg/m^3)	2000.0	1600.0
V_f	0.6	0.6

The designer must take into account the factor of safety when designing a structure. Since, composites are highly orthotropic and their fractures were not fully studied the factor of safety was taken as 2.

4.2. Torque Transmission of the Composite drive Shaft

4.2.1. Stress-Strain Relationship for Unidirectional Lamina

The lamina is thin and if no out-of-plane loads are applied, it is considered as the plane stress problem. Hence, it is possible to reduce the 3-D problem into 2-D problem. For unidirectional 2-D lamina, the stress-strain relationship in terms of principal material directions is given by:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}, \qquad (7)$$

where σ, τ, γ and ε represent stresses and strains in material directions. The matrix Q is referred to as the reduced stiffness matrix for the layer and its terms are given by:

$$Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}} \; ; \qquad Q_{12} = \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} \; ;$$

$$Q_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}} \; ; \qquad Q_{66} = G_{12} \, .$$

4.2.2. Stress strain relation in arbitrary orientation

For an angle-ply lamina, where fibers are oriented at an angle with the positive X-axis (longitudinal axis of shaft), the stress strain relationship is given by:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}, \qquad (8)$$

where σ and ε represent normal stresses and strains in X, Y and XY dirctions respectively and bar over Q_{ij} matrix denotes transformed reduced stiff nesses. Its terms are individually given by:

$$\overline{Q_{11}} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \, ;$$

$$\overline{Q_{12}} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4) \, ;$$

$$\overline{Q_{16}} = (Q_{11} - Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})cs^3 \, ;$$

$$\overline{Q_{22}} = Q_{11}S^4 + Q_{22}S^4 + 2(Q_{11} + 2Q_{66})S^2C^2 \, ;$$

$$\overline{Q_{26}} = (Q_{11} - Q_{12} - 2Q_{66})CS^3 - (Q_{22} - Q_{12} - 2Q_{66})C^3S \, ;$$

$$\overline{Q_{66}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4) \, ;$$

with $C = \cos\theta$ and $S = \sin\theta$.

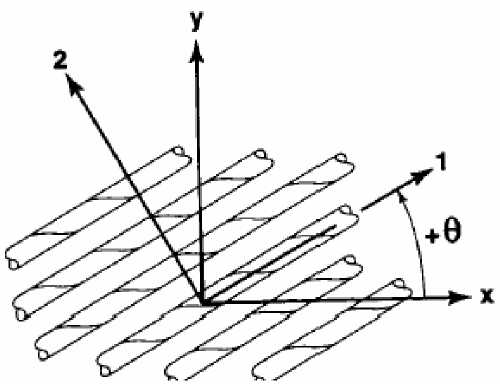

Fig. 2. Shows relation between material coordinate system and X – Y coordinate system

4.2.3. Force and moment resultants

For a symmetric laminate, the B matrix vanishes and the in plane and bending stiffnesses are uncoupled.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix}; \qquad (9)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix}, \qquad (10)$$

where N_x, N_y, N_{xy} and M_x, M_y, M_{xy} in (9), (10) referred as forces and moments per unit width.

$$A_{ij} = \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}) \, ; \qquad (11\text{ a})$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2) ; \quad (11\text{ b})$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3) , \quad (11\text{c})$$

where represent A_{ij}, B_{ij} and D_{ij} are extensional, coupling and bending stiffnesses having $i,j = 1, 2...6$ respectively, h_k is the distance between the neutral fiber to the top of the K^{th} layer.

Strains in the reference surface is given by:

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}, \quad (12)$$

where

$$\begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1} .$$

4.2.4. Elastic constants for the composite shaft

Elastic constants for the composite shaft are given by:

$$E_x = \frac{1}{t}\left[A_{11} - \frac{A_{12}^2}{A_{22}} \right]; \qquad E_y = \frac{1}{t}\left[A_{22} - \frac{A_{12}^2}{A_{11}} \right] ,$$

where E_x and E_y are the Young's modulus of the shaft in axial and hoop direction:

$$G_{xy} = \frac{A_{66}}{t} ; \qquad \nu_{xy} = \frac{A_{12}}{A_{11}} ,$$

where G_{xy} and ν_{xy} are the rigidity modulus in xy plane and Poisson's ratio of the composite shaft.

When a shaft is subjected to torque T, the resultant forces N_x, N_y, N_{xy} in the laminate by considering the effect of centrifugal forces are:

$$N_x = 0; \quad N_y = 2\rho t r^2 \omega^2; \quad N_{xy} = \frac{T}{2\pi r^2} , \quad (13)$$

where ρ is the density, t is the thickness, r mean radius and ω is the angular velocity of the composite shaft knowing the stresses in each ply, the failure of the laminate is determined using the first ply failure criteria. That is, the laminate is assumed to fail when the first ply fails. Here maximum stress theory is used to find the torque transmitting capacity

4.3. Torsional Buckling Capacity

Since long thin hollow shafts are vulnerable to torsional buckling, the possibility of the torsional buckling of the composite shaft was checked by the expression for the torsional buckling load T_{cr} of a thin walled orthotropic tube, which was expressed below:

$$T_{cr} = (2\pi r^2 t)(0.272)(E_x E_y{}^3)^{0.25} (t / r)^{1.5} , \quad (14)$$

where E_x and E_y are the Young's modulus of the composite shaft in axial and hoop direction, r and t are the mean radius and thickness of the composite shaft.

This equation has been generated from the equation of isotropic cylindrical shell and has been used for the design of drive shafts. From the equation (14), the torsional buckling capability of composite shaft is strongly dependent on the thickness of composite shaft and the average modulus in the hoop direction.

4.4. Lateral Vibration

Natural frequency f_{nt} based on the Timoshenko beam theory is given by:

$$f_{nt} = K_s \frac{30\pi p^2}{L^2} \sqrt{\frac{E_x r^2}{2\rho}} ; \quad (15)$$

$$\frac{1}{K^2{}_s} = 1 + \frac{p^2 \pi^2 r^2}{2L^2}\left[1 + \frac{f_s E_x}{G_{xy}} \right],$$

where f_{nt} and p are the natural and first natural frequency. K_s is the shear coefficient of the natural frequency (< 1), f_s is a shape factor (equals to 2) for hollow circular cross-sections.

Critical speed:

$$N_{crt} = 60 f_{nt} . \quad (16)$$

5. DESIGN OPTIMIZATION

Most of the methods used for design optimization assume that the design variables are continuous. In structural optimization, almost all design variables are discrete. A simple GA is used to obtain the optimal number of layers, thickness of ply and fiber orientation of each layer. All the design variables are discrete in nature and easily handled by GA. With reference to the middle plane, symmetrical fiber orientations are adopted.

5.1. Comparison between GA and other methods

GA differs from traditional optimization algorithm in many ways. A few are listed here [7]:

- GA does not require a problem specific knowledge to carry out a search. GA uses only the values of the objective function. For instance, calculus based search algorithms use derivative information to carry out a search;
- GA uses a population of points at a time in contrast to the single point approach by the traditional optimization methods. That means at the same time GAs process a number of designs.

5.2. Comparison between biological GA terms

Chromosome – a small rod like body found in the living cells, which is responsible for the transmission of generic information denotes coded design vector in GA.

Gene – which is a part of the chromosome carrying the hereditary information denotes each bit in the coded design vector in GA.

Population – denotes a number of coded design variables in a cell, where as G*eneration* denotes the population of design vectors, which are obtained after one computation in other words the process of termination of the loop was carried out by fixing the maximum number generations. This maximum number generations is fixed after trail runs.

5.3. Objective Function

The objective for the optimum design of the composite drive shaft is the minimization of weight, so the objective function of the problem is given as weight of the shaft:

$m = \rho AL$

or

$$m = \rho \frac{\pi}{4}\left(d_o^2 - d_i^2\right)L \,. \tag{17}$$

5.4. Design Variables

The design variables of the problem are

- Number of plies $[n]$;
- Stacking Sequence $[\theta_k]$;
- Thickness of the ply $[t_k]$.

The limiting values of the design variables are

- $n \ge 0$;
- $-90 \le \theta_k \le 90$;
- $0.1 \le t_k \le 0.5$,

where $k = 1, 2, \ldots, n$ and $n = 1, 2, 3, \ldots, 32$.

The number of plies required depends on the design constraints, allowable material properties, thickness of plies and stacking sequence. Based on the investigations it was found that up to 32 numbers of plies are sufficient.

5.5. Design Constraints

1. Torque transmission capacity of the shaft $T \ge T_{\max}$.
2. Bucking torque capacity of the shaft $T_{cr} \ge T_{\max}$.
3. Lateral fundamental natural frequency $N_{crt} \ge N_{\max}$.

The constraint equations may be written as:

$$C_1 = \left(1 - \frac{T}{T_{\max}}\right) \quad \text{If } T < T_{\max}$$
$$= 0 \text{ Otherwise}$$

$$C_2 = \left(1 - \frac{T_{cr}}{T_{\max}}\right) \quad \text{If } T_{cr} < T_{\max}$$
$$= 0 \text{ Otherwise}$$

$$C_3 = \left(1 - \frac{N_{crt}}{N_{\max}}\right) \quad \text{If } N_{crt} < N_{\max}$$
$$= 0 \text{ Otherwise}$$

$$C = C_1 + C_2 + C_3 \,.$$

Using the method of Rajeev and Krishnamoorthy [13], the constrained optimization can be converted to unconstrained optimization by modifying the objective function as:

$$\phi = m(1 + k_1 C) \tag{18}$$

For all practical puroses, k_1 is a penalty constant and is assumed to be 10.

5.6. Input GA parameters

Input GA parameters of E-glass / epoxy and and HM carbon/epoxy composite drive shafts are shown in Table 3. Here symmetric laminates are considered.

Table 3. Input GA parameters

Number of parameters	$n/2 + 2$ if n is even $(n+1)/2 + 2$ if n is odd
Total string length	139
Population size	50
Maximum generations	150
Cross-over probability	1
Mutation probability	0.003
String length for number of plies	5
String length for fiber orientation	8
String length for thickness of ply	6

Total string length is equal to the string length for number of plies plus 16 string length for fiber orientation and plus the string length for thickness of ply. In total it equals to 139.

6. COMPUTER PROGRAM

An attempt has been made to develop a powerful and efficient computer program using C language to perform the optimization process, and to obtain the best possible design. The flow-chart describing the step-by-step procedure of optimizing the composite drive shaft using GA is shown in Fig. 4.

7. RESULTS AND DISCUSSION

7.1. GA results of E-glass / epoxy drive shaft

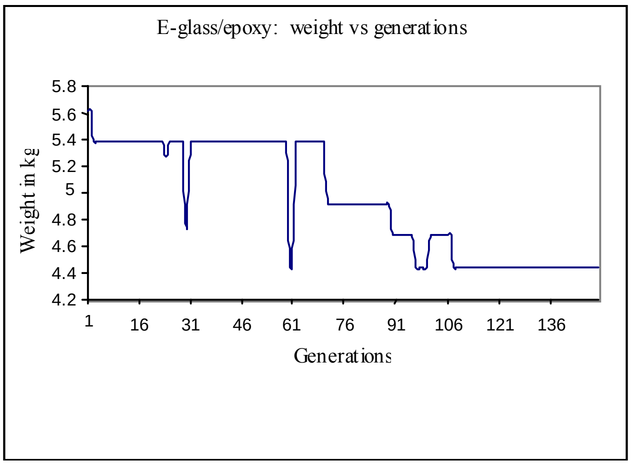

Fig. 3. Variation of weight of E-glass / epoxy drive shaft with number of generations

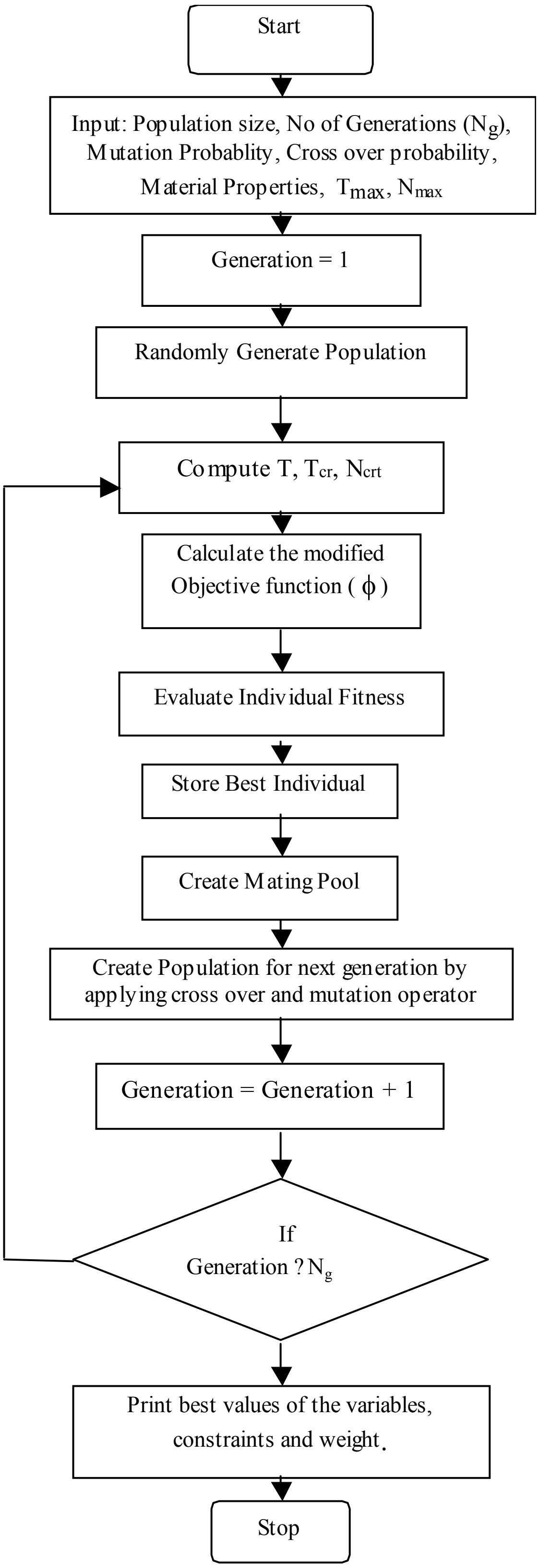

Fig. 4. Flow chart of GA based optimal design

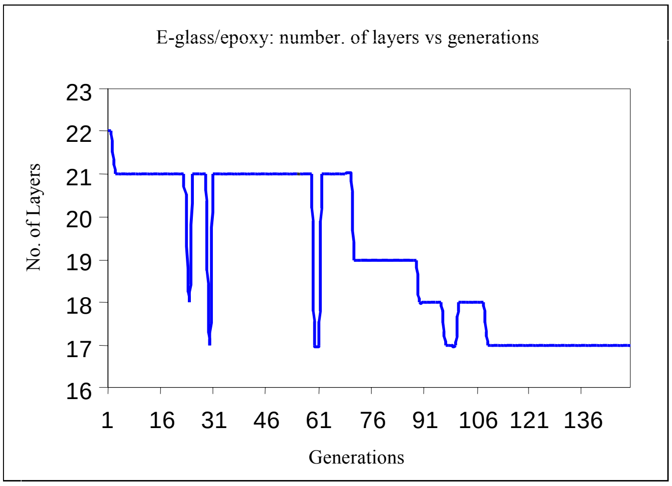

Fig. 5. Variation of number of layers of E-glass / epoxy drive shaft with number of generations

7.2. GA results of HM carbon / epoxy drive shaft

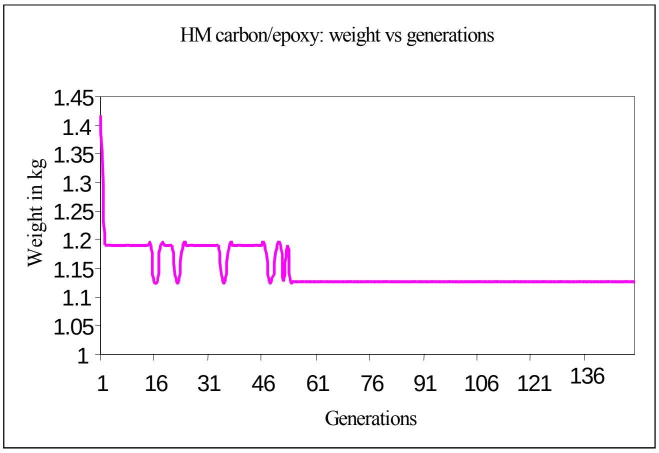

Fig. 6. Variation of the weight of HM carbon / epoxy drive shaft with number of generations

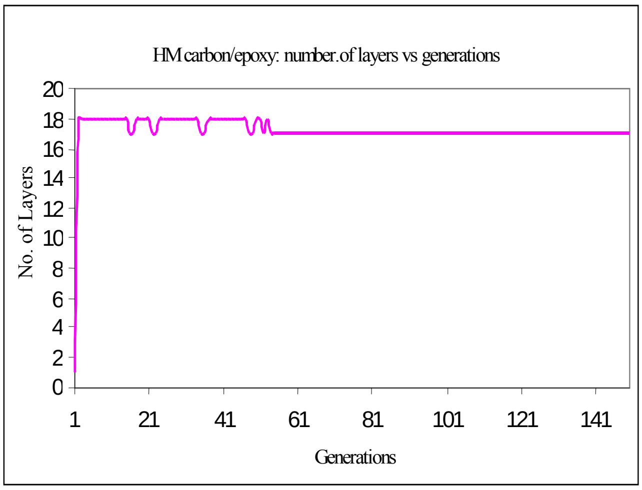

Fig. 7. Variation of number of layers of HM carbon / epoxy drive shaft with number of generation

8. SUMMARY OF GA RESULTS

Table 4. Optimal design values of steel and composite drive shaft (tube only)

	d_o (mm)	L (mm)	t_K (mm)	N optimal layers	t (mm)	Optimum Stacking sequence	T (Nm)	T_{cr} (Nm)	N_{crt} (rpm)	Wt (kg)	$Wt.$ saving (%)
Steel	90	1250	3.32	1	3.32	------	3501.9	43857.9	9323.7	8/60	----
E-glass/ /epoxy	90	1250	0.4	17	6.8	$[46/–64/–15/–13/39/ /–84/–28/20/-\overline{27}]_s$	3525.4	29856.5	6514.6	4.44	48.36
HM carbon/ /epoxy	90	1250	0.12	17	2.04	$[–65/25/68/–63/36/ /–40/–39/74/-\overline{39}]_s$	3656.7	3765.8	9270.3	1.13	86.90

9. CONCLUSIONS

1. A procedure to design a composite drive shaft is suggested**.**

2. Drive shaft made up of E-glass / epoxy and high modulus carbon / epoxy multilayered composites have been designed.

3. The designed drive shafts are optimized using GA for better stacking sequence, better torque transmission capacity and bending vibration characteristics.

4. The usage of composite materials and optimization techniques has resulted in considerable amount of weight saving in the range of 48 to 86 % when compared to conventional steel shaft.

5. These results are encouraging and suggest that GA can be used effectively and efficiently in other complex and realistic designs often encountered in engineering applications.

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