

The Effect of Stochastically Dependent Physical Parameters on the Materials' Thermal Receptivity Coefficient

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The most frequently employed calculation methods to determine the thermal receptivity of multi-layer partitions do not allow the correct estimation of the thermo-physical parameters of lightweight timber frame partitions. The mentioned assertion was examined and proved on the basis of the results achieved both by the use of computer simulation tools and during the experimental research. In fact, the parameters defining the thermo-physical properties of the materials calculated by the introduced methodology differed from the ones achieved by experiment up to 40 %. The effect of the stochastically dependent physical parameters on the thermal receptivity coefficients of the materials of the lightweight timber frame partition is discussed in this paper. The calculations based on the experiment data were performed by adopting the methods of mathematical statistics and the theory of chances.

Keywords: stochastic parameters, thermal receptivity, thermal conductivity, statistics, the lightweight timber frame partition.

1. INTRODUCTION

Number of the thermo physical parameters (heat flux density, thermal inertia, time constant, thermal resistance, time lag, decrement factor and etc.) describes the thermal behavior of building envelope. Usually these parameters are stochastically dependent because the building envelope is heterogeneous.

Precision in the calculation of the mentioned thermo physical parameters depends on the precision of the separate material's physical parameters (thermal conductivity, density, thickness, specific heat capacity and etc.) and their interdependence. The precision is worked out by employing the methods of the mathematical statistics [1].

The review of the scientific publications revealed [2–4], that the precise estimation of the variable parameters can be assured by the corresponding experimental input data (comprehensive temperature field and moisture content inside the building envelope). Such an experimental input data [5] was used to investigate the effect of stochastically dependent physical parameters (λ , c , ρ) [6, 7] on the materials' thermal receptivity coefficient s .

The material's thermal receptivity coefficient s is used for the calculation of the heat losses through partitions by choosing the outside temperature [8].

It is defined by the following equation:

$$s = \sqrt{\frac{2\pi\lambda c\rho}{z}}; \text{ W/(m}^2\text{K)}, \quad (1)$$

where λ denotes thermal conductivity of a material, W/(mK); c points to specific heat capacity of a material, J/(kgK); ρ denotes density of a material, kg/m³; z is oscillation period of outside air temperature, hours.

Thus, the thermal receptivity coefficient depends on the physical properties of materials and the oscillation period of outside air temperature. As the oscillation is most frequently considered to occur in a one-day period, the equation (1) can be simplified in the following way:

$$s = 0.51\sqrt{\lambda c\rho} . \quad (2)$$

The coefficient s is also found in the Standard [9] where it is used for the estimation of thermal resistance in external partitions. A. Lykov [10] maintained that the coefficient s does not evaluate the material's ability to either accumulate or release the heat during the periodical temperature oscillation that takes place on the surface of the body or in the environment. Therefore doubts arise, whether the theoretical equations estimating material's thermal receptivity coefficient are still reasonable and reflect the real heat amounts accumulated or released by the materials.

The parameter estimating material's thermal receptivity can be found in European Standards as well [11], where it is determined as effective heat capacity C , J/K depending on the material's density, specific heat capacity, and the area of the partition. Here effective heat capacity C is used for the estimation of the time constant, which specifies thermal inertia of the heated spaces.

The difference between s and C is that s is used directly in the equations for calculation of such a parameters as temperature wave time lag and decrement factor [12]. Meanwhile, C is used to evaluate the dynamic of the temperature distribution and heat flux inside building envelope. Other thermo physical parameters come out from these main items.

The material's thermal receptivity coefficient s is fully explored for heavyweight building envelope [12], [13] during the hot exploitation period. The thermal behavior analysis of the lightweight timber frame walls on the climatic conditions of hot period in Eastern Europe was not found in the available scientific publications databases. So,

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the paper aims at the precise calculation of the thermal receptivity coefficient of the lightweight timber frame wall materials during the hot exploitation period. The timber frame wall with a rather complex structure yet nevertheless most frequently used in practice was chosen for the investigation (Fig. 1).

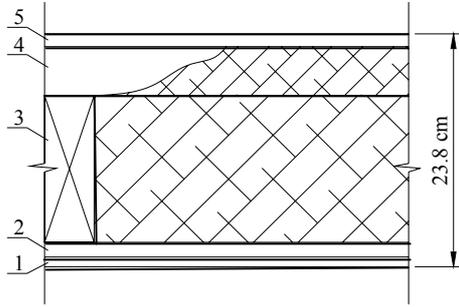


Fig. 1. The cross section of the tested wall 1 – plaster, $d = 0.01$ m; 2 – OSB (Oriented Strandboard), $d = 0.015$ m; 3 – timber frame (150×50 mm, step 600 mm) with mineral wool inside; 4 – cross beam (50×50 mm, step 600 mm) with mineral wool inside; 5 – gypsum plasterboard, $d = 0.013$ m

2. THE MATHEMATICAL MODEL FOR THE ESTIMATION OF THE MATERIALS' THERMAL RECEPTIVITY COEFFICIENT

The materials of which the tested wall was composed combine a wide range of physical parameters. They are presented in Table 1.

Table 1. The physical parameters of the tested materials

Materials	Thickness of the layer d , m	Thermal conductivity λ , W/(mK)	Density ρ , kg/m ³	Specific heat capacity c , J/(kgK)
Plaster	0.01	0.8	1700	1100
OSB	0.015	0.08	900	1700
Mineral wool 1	0.05	0.036	100	840
Gypsum plasterboard	0.013	0.22	800	1050
Mineral wool 2	0.15	0.039	40	840
Timber stud	0.15	0.18	500	1610

Before calculating the average value of the material's thermal receptivity coefficient it is necessary to estimate the designed values of its physical parameters. Since the construction is tested during the hot period of the year it must be considered that the excess moisture has already evaporated from the materials during summer and it does not affect the declared values of specific heat capacity and density. That is why these parameters are set as constants.

The declared values of the thermal conductivity coefficient with the 90 % quantile and 90 % 1-tailed confidence interval must be maintained by producers (Fig. 2) [14]. The mean value of the sample's temperature should not exceed 10 °C and the settled moisture saturation should be estimated under 23 °C of temperature and 50 % of relative humidity.

As the wall was tested under the natural exploitation conditions, the declared values of the materials' thermal

conductivity were defined by the method presented in the Standard [14].

Consider:

$$\lambda_{dec} = \lambda F_T F_m; \quad (3)$$

where λ defines the actual thermal conductivity coefficient determined by testing the material W/(mK), which, in the discussed case, is a stochastically dependent parameter; F_T is temperature conversion factor; F_m is humidity conversion factor.

Having estimated the temperature and humidity conversion factors and their descriptive statistics (mean values, standard deviations, variances, etc.) it was assumed, that the error of thermal conductivity coefficient did not increase significantly under the mentioned conditions. Therefore temperature and humidity conversion factors were set as constants ($F_T = \text{const}$, $F_m = \text{const}$). Here the designed value of the material's thermal conductivity coefficient is defined by the following equation [14]:

$$\lambda_{ds} = \lambda_{dec} + \Delta\lambda_{\omega} + \Delta\lambda_{CV} = \lambda_{dec} + \Delta\lambda_{\omega} + \Delta\lambda_{dec} K_{CV}, \quad (4)$$

where λ_{dec} defines the declared value of the material's thermal conductivity W/(mK); $\Delta\lambda_{\omega}$ points to correction because of the material's additional humidity inside the structure W/(mK); $\Delta\lambda_{CV}$ is the correction because of thermal convection W/(mK); K_{CV} defines the thermal convection coefficient.

In the analyzed case, $\Delta\lambda_{\omega}$ values twice lower than the ones presented in Standard [14] will be used, since the error $\Delta\lambda_{\omega}$ was determined for the winter period.

According to the use of the expression (3), the equation (4) can be rearranged in order to find out the descriptive characteristics:

$$\begin{aligned} \lambda_{ds} &= \lambda F_T F_m + \lambda F_T F_m K_{CV} + \Delta\lambda_{\omega} = \\ &= \lambda [(F_T F_m (1 + K_{CV}))] + \Delta\lambda_{\omega} = \lambda A + \Delta\lambda_{\omega}; \end{aligned} \quad (5)$$

Then the mean value of the designed thermal conductivity coefficient can be defined as follows:

$$\bar{\lambda}_{ds} = A \bar{\lambda} + \Delta\lambda_{\omega} \quad (6)$$

The mean value of the actual thermal conductivity coefficient $\bar{\lambda}$ can be defined by the given equation [15]:

$$\bar{\lambda} = \lambda_{0.90} - 1.3\sigma\lambda \quad (7)$$

where $\lambda_{0.90}$ is thermal conductivity coefficient with 90 % quantile (the declared value is taken from the technical specifications or manuals) W/(mK); 1.3 is the coefficient estimating the amount of sample values found in the statistics manuals [16]; $\sigma\lambda$ defines standard deviation of the thermal conductivity coefficient W/(mK).

Since the value $\sigma\lambda$ comes out to be the commercial secret of every manufacturer, the authors should only refer to the results of the independent laboratory research. Supposing that the distribution of the material's thermal conductivity coefficient $p(\lambda)$ is normal, the hatched area in Fig. 2 will illustrate the probability of the declared values of the thermal conductivity coefficient entering there the probability being not less than 90 %.

The variance of the material's thermal conductivity coefficient can be calculated since its Standard deviation is $\sigma\lambda$ [1]:

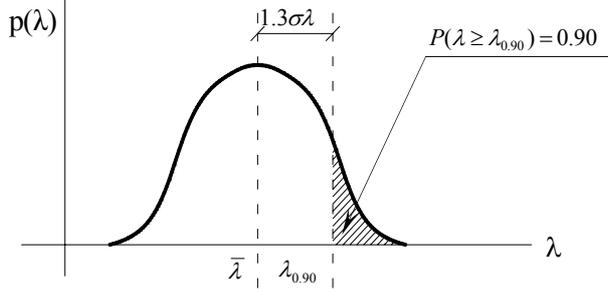


Fig. 2. Geometrical interpretation of the thermal conductivity coefficient distribution

$$\sigma^2 \lambda = (\sigma\lambda)^2 . \quad (8)$$

The variance of the designed thermal conductivity coefficient can be estimated by the differentiation of the equation (6):

$$\sigma^2 \lambda_{ds} = A^2 \sigma^2 \lambda . \quad (9)$$

Now it is possible to proceed to the estimation of the confidence interval of the material's thermal receptivity coefficient, as the descriptive statistics of the stochastically dependent parameter are already known.

The theoretical thermal receptivity coefficient of the material is estimated by the equation (1). The mean value of the tested materials' designed thermal conductivity coefficient can be defined as follows [1]:

$$\bar{s}_{ds} = \sqrt{\frac{2\pi c \rho \bar{\lambda}_{ds}}{z}} = B \sqrt{\bar{\lambda}_{ds}} , \quad (10)$$

where the constant factor B is introduced.

Consider:

$$B = \sqrt{\frac{2\pi c \rho}{z}} , \quad (11)$$

The variance of the designed thermal receptivity coefficient can be estimated by the differentiation of the equation (10) by λ_{ds} :

$$\sigma^2 s_{ds} = \frac{B^2}{\lambda_{ds}} \sigma^2 \lambda_{ds} . \quad (12)$$

The required standard deviation of the designed thermal receptivity coefficient is found out by the given formula:

$$\sigma s_{ds} = \sqrt{\sigma^2 s_{ds}} . \quad (13)$$

Now the material's thermal receptivity coefficient with the 90 % quantile and 90 % 1-tailed confidence interval [1] can be defined as follows:

$$s_{ds,0.90} = \bar{s}_{ds} + 1.3\sigma s_{ds} . \quad (14)$$

3. CALCULATION OF THERMAL RECEPTIVITY COEFFICIENT ON THE BASIS OF THE EXPERIMENTAL DATA

The materials' thermal receptivity coefficients of the described tested wall (Fig. 1) have been calculated with the

use of the mathematical model presented in Section 2 and the parameters obtained experimentally [5]. The experiment data embraces the comprehensive temperature field at the cross cut of the tested wall and the declared values of the materials' thermal parameters. First of all, the temperature conversion factor F_T is calculated according to the mean temperatures of each layer [14]. Then the humidity conversion factor F_m [14] is determined according to the settled level of moisture saturation. The conversion factors for each material of the tested wall construction are shown in Table 2:

Table 2. The conversion factors for each material

Materials	Temperature conversion factor F_T	Humidity conversion factor F_m
Plaster	1.002	1.271
OSB	1.002	1.271
Mineral wool 1	1.053	1.000
Gypsum plasterboard	1.043	1.197
Mineral wool 2	1.034	1.000
Timber stud	1.010	1.271

Another important parameter that is usually determined in laboratories of thermal building physics by testing the materials' thermal conductivity is the standard deviation of the thermal conductivity coefficient $\sigma\lambda$. The values of $\sigma\lambda$ that were used in the calculations were measured in Laboratory of Thermal Building Physics and are presented in Table 3.

The mean actual values of the thermal conductivity coefficient $\bar{\lambda}$ have been calculated by using the $\sigma\lambda$ values in equation (7). The mean designed values of the thermal conductivity coefficient, in their turn, have been defined by the equation (6). It should be noted that no heat transfer by convection takes place in the tested wall due to efficient consolidation between the combined materials. Thus, for all the materials, the thermal convection coefficient K_{CV} used in the equation (5) is equal to zero.

The variances of the actual and designed thermal conductivity coefficient values have been estimated by the equations (8) and (9). The calculated descriptive statistics of the thermal conductivity coefficient are presented in Table 3.

Table 3. Descriptive statistics of the thermal conductivity coefficient

Materials	Descriptive statistics of materials' thermal conductivity coefficient				
	$\sigma\lambda$, W/(mK)	$\bar{\lambda}$, W/(mK)	$\bar{\lambda}_{ds}$, W/(mK)	$\sigma^2\lambda$, (W/(mK)) ²	$\sigma^2\lambda_{ds}$, (W/(mK)) ²
Plaster	0.00300	0.7961	1.029	9.000E-06	0.00001459
OSB	0.00080	0.0790	0.116	6.400E-07	0.00000104
Mineral wool 1	0.00050	0.0354	0.038	2.500E-07	0.00000028
Gypsum plasterboard	0.00200	0.2174	0.281	4.000E-06	0.00000623

Mineral wool 2	0.00046	0.0384	0.041	2.116E-07	0.00000023
Timber stud	0.00150	0.1781	0.244	2.250E-06	0.00000371

As Table 3 illustrates, plaster has the highest value of the variance of the designed thermal conductivity coefficient. It could be caused by an unequal distribution of the plaster's components, by different velocity of the compounds' consolidation and other factors. What is more, the plaster compound is finally prepared for usage at the site, when all the other materials of which the tested wall consists are finally moulded at the factories where the allowable errors of the thermal parameters are strictly controlled. Thus, plaster should cause the biggest errors in the calculation of the physical parameters for the entire wall.

The mean values and the deviations of the designed thermal receptivity coefficient can be estimated with descriptive statistics of the stochastically dependent parameter λ . First of all the mean values of the designed thermal receptivity coefficient \bar{s}_{ds} according to the equation (10) are calculated. Then the constant factor B according to the equation (11) is calculated using the declared values of the materials' density and specific heat capacity [6, 7]. These parameters are accepted as constants ($\rho = \text{const}$, $c = \text{const}$) because the excess moisture has evaporated from the materials during summer. The assumed period of the outside air temperature oscillation is equal to 24 hours.

The variance of the designed thermal receptivity coefficient $\sigma^2 s_{ds}$ is calculated by installing the constant factor B and the variance of the designed thermal conductivity coefficient $\sigma^2 \lambda_{ds}$ into equation (12).

The standard deviation of the designed thermal receptivity coefficient, in its turn, is determined by equation (13).

The last step is the calculation of the designed values of the materials' thermal receptivity coefficient $s_{ds,0.90}$ with the probability 0.90 according to the equation (14). The cases of the calculated descriptive statistics of the designed thermal receptivity coefficient are presented in Table 4.

Table 4. Descriptive statistics of the designed thermal receptivity coefficient

Materials	Descriptive statistics of the designed thermal receptivity coefficient			
	\bar{s}_{ds} , W/(m ² K)	$\sigma^2 s_{ds}$ (W/(m ² K)) ²	σs W/(m ² K)	$s_{ds,0.90}$, W/(m ² K)
Plaster	11.83	0.00193	0.04392	11.89
OSB	3.59	0.00100	0.03161	3.63
Mineral wool 1	0.48	0.00004	0.00666	0.49
Gypsum plasterboard	4.15	0.00135	0.03678	4.19
Mineral wool 2	0.32	0.00001	0.00369	0.32
Timber stud	3.78	0.00089	0.02986	3.82

According to Table 4, the greatest standard deviations of the designed thermal receptivity coefficient were for the plaster and gypsum plasterboard (i.e. 4.39 % and 3.67 %).

The smallest errors were determined for thermal insulation materials.

Table 5 presents the designed values of the thermal receptivity coefficient s_{ds} taken from Standard [8] and they relative errors from the calculated ones $s_{ds,0.90}$:

Table 5. The designed values of the thermal receptivity coefficient

Materials	s_{ds} , W/(m ² K)	$s_{ds,0.90}$, W/(m ² K)	Relative error, %
Plaster	11.00	11.89	3.87
OSB	4.50	3.63	10.74
Mineral wool 1	0.47	0.49	2.28
Gypsum plasterboard	3.70	4.19	6.25
Mineral wool 2	0.27	0.32	8.50
Timber stud	3.20	3.82	8.77

The results presented in Table 5 show that the greatest difference between the standard and the calculated values was defined for the timber based materials with high accumulative properties and relatively low thermal conductivity (see Table 1). However, the obtained differences between the thermal receptivity coefficients mentioned above (see Table 5) were caused by the exploitation conditions of the materials and the interaction between them in the whole structure rather than by the combinations of physical parameters.

4. CONCLUSIONS

1. Thermal convection did not affect the designed thermal conductivity coefficients of the materials of the tested structure. Only the correction of the material's additional humidity inside the wall $\Delta\lambda_w$, which in a hot period is selected according to the settled humidity, has been estimated.
2. The temperature F_T and humidity F_m conversion factors did not make any significant effect on the precision of the designed thermal conductivity coefficient during the exploitation of the structure and therefore were set as constants ($F_T = \text{const}$, $F_m = \text{const}$).
3. The investigation demonstrated that it is necessary to work out the database of the standard deviation of the thermal conductivity coefficient $\sigma\lambda$ for each building material under the measurements obtained in the independent laboratories.
4. As it has been clarified during the analysis of descriptive statistics, insignificant standard deviations of the thermal conductivity coefficient can cause considerable standard deviations of the thermal receptivity coefficient.
5. The standard deviations of the tested materials' thermal receptivity coefficients were not inadmissible (max. 4.39 %), nevertheless, the effect of these errors should be estimated in further calculations.
6. Significant errors of the timber frame wall materials' thermal receptivity coefficient were obtained because of the different exploitation conditions during the hot and cold periods of the year. That is why the authors recommend to estimate the conversion factors of the materials' thermal receptivity coefficient for different

exploitation seasons and for partitions with the varying thermal inertia.

LIST OF SYMBOLS

s – material’s thermal receptivity coefficient, $W/(m^2K)$;
 λ – thermal conductivity of a material, $W/(mK)$;
 c – specific heat capacity of a material, $J/(kgK)$;
 ρ – density of a material, kg/m^3 ;
 z – oscillation period of outside air temperature, hours;
 C – effective heat capacity, J/K ;
 d – thickness of the layer, m;
 F_T – temperature conversion factor, –;
 F_m – humidity conversion factor, –;
 λ_{ds} – the designed value of the thermal conductivity of a material, $W/(mK)$;
 λ_{dec} – the declared value of the thermal conductivity of a material, $W/(mK)$;
 $\Delta\lambda_w$ – correction because of the material’s additional humidity inside the structure, $W/(mK)$;
 $\Delta\lambda_{CV}$ – correction because of thermal convection, $W/(mK)$;
 K_{CV} – the thermal convection coefficient, –;
 $\bar{\lambda}_{ds}$ – the mean value of the designed thermal conductivity coefficient of a material, $W/(mK)$;
 $\bar{\lambda}$ – the mean value of the actual thermal conductivity coefficient of a material, $W/(mK)$;
 $\lambda_{0,90}$ – thermal conductivity coefficient of a material with 90% quantile, $W/(mK)$;
 $\sigma\lambda$ – standard deviation of the thermal conductivity coefficient of a material, $W/(mK)$;
 $\sigma^2\lambda$ – variance of the thermal conductivity coefficient of a material, $(W/(mK))^2$;
 $\sigma^2\lambda_{ds}$ – variance of the designed thermal conductivity coefficient of a material, $(W/(mK))^2$;
 \bar{s}_{ds} – the mean value of the designed thermal receptivity coefficient of a material, $W/(m^2K)$;
 σ^2s_{ds} – variance of the designed thermal receptivity coefficient of a material, $(W/(m^2K))^2$;
 σs_{ds} – standard deviation of the designed thermal receptivity coefficient of a material, $W/(m^2K)$;
 $s_{ds, 0,90}$ – material’s thermal receptivity coefficient, with the 90 % quantile, $W/(m^2K)$.

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