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# Simulation of Roughened Surfaces

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In the various fields of adhesive bonding mechanical surface roughening by abrasion is used to increase real surface area of the substrate. Mathematically, obtained surface roughness can be treated as realization of a Gauss process. If the correlation function of the profile is known, spectral density can be determined, also. It is shown that relation exist between abrasion tool surface profile and abraded surface profile of soft polymer material. Relations between correlation functions of both these surfaces are determined. During simulation of abraded profiles, it was determined that used simulation algorithm describes the abrasion process.

*Keywords*: surface roughness, correlation function, profile.

## INTRODUCTION

In the various fields of adhesive bonding mechanical surface treatment of substrates by abrasion is used not only to remove impurities from the surface, but to increase real surface area, also. That results on the increase of real contact area and in some cases strength of adhesives joints. The degree of roughness is important design factor for the adhesive bonds of soft polymeric materials.

After abrasion the roughened surface is characterised by individual parameters. As the surface profile can be interpreted as realization of normal stationary process, main criteria for its approximation is a correlation function, which allows to obtain spectral density of the profile [1 – 3]. Besides, relation between spectral densities of the interacting surfaces exists. So, it is important mathematically to describe interaction between two surfaces during abrasion treatment.

The simulation of roughened surfaces is important task, also. There are several methods, which allow to simulate profiles of real roughened surfaces: method of Monte Carlo, based on the random generation of numbers [4 – 5] or stochastic integral equation method [6 – 7]. The last one contains some advantages as compared to those of Monte Carlo, such as obviation of large amount of iterations, which are needed during the Monte Carlo simulation and unnecessary discreetization of roughened surface. One more method is based on the fractal simulation and is known as a surface growth method [8 – 9]. Main parameter of this simulation is surface fractal dimension. There a large amount of iterations up to surface forming – the same as for Monte Carlo method is needed. Another method, so called bidirectional reflectance distribution function (BRDF) is based on the interpolation of surfaces images [10 – 11]. All of these methods are possible for both 2D and 3D views.

In this paper method and algorithm for simulation of roughened surfaces and roughened surface changes during abrasion treatment based on parabolas simulation are presented.

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## THEORY

Early it was determined relation between abrasion tool and abraded rubber surface roughness after this kind of mechanical treatment [2 – 3]. It was determined, that roughness of both abrasion tool and abraded rubber can be assumed as random normal process. So, profiles carried out from these surfaces can be treated as realization $N(m,\sigma)$ of this process [1 – 3], which can be approximated according to the following correlation function:

$$K(\tau)=\frac{C}{1+(\alpha\tau)^2}\ , \tag{1}$$

where $C$, $\alpha$ are the constants determined by the method of least squares.

Then spectral density of profile is:

$$s(\omega)=\frac{C}{2\alpha}\exp\left(-\frac{|\omega|}{\alpha}\right), \tag{2}$$

where $\omega$ is the frequency of the process.

The relation between investigated surfaces spectral densities were determined also [1 – 3].

Process of soft polymer material surface roughening can be assumed as linear transformation of abrasive profile $X(t)$ to the soft polymer material surface profile $X^*(t)$. In this case this equation is correct:

$$s^*(\omega)=\left|\Phi(i\omega)\right|^2 s(\omega)\,, \tag{3}$$

where $s^*(\omega)$ is the spectral density of abraded soft rubber surface, $\Phi(i\omega)$ is the iteration parameter of the process, expressed as:

$$\left|\Phi(i\omega)\right|^2=\frac{\left|b_1 i\omega+b_0\right|}{\left|a_1 i\omega+a_0\right|}=\frac{b_1^2\omega^2+b_0^2}{a_1^2\omega^2+a_0^2}\,, \tag{4}$$

where $a_1$ and $b_1$ are experimentally determined parameters.

Performed investigations were related to decrease of time-consuming operations during which large amount of profiles (of both investigated surfaces) are obtained. Eq. 1 – 3 was used to simulate part of them. Firstly, algorithm presented in the Fig. 1 was constructed and appropriate profiles were simulated.

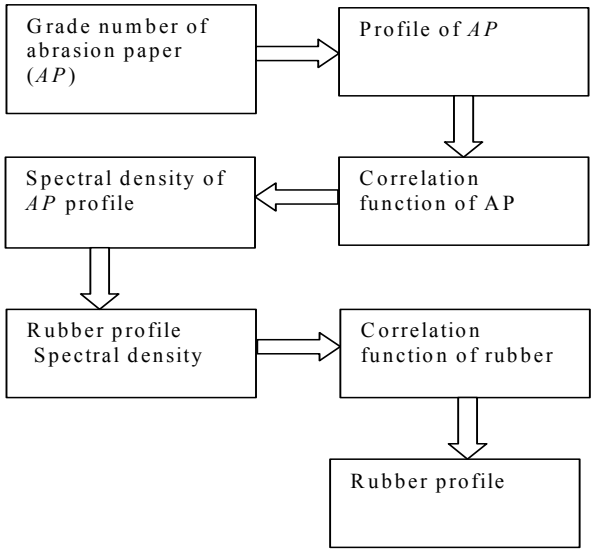


**Fig. 1.** Algorithm of the relation between abrasion paper and rubber surface profiles

Simulation of the profiles is based on two assumptions:

- profiles are realizations of the stationary normal process $\boldsymbol{N}(m, \sigma)$ wich characterises two parameters: average $m$ and dispersion $\sigma^2$ [1 – 2];
- independently of the surface nature its roughness can be approximated by parabolas [3].

The simulation was divided in to following stages:

1) generation of three random points $y0, y1, y2$ distributed according to the $\boldsymbol{N}(m,\sigma)$ (Fig. 2);
2) calculation of coefficients of the parabola equation $y = ax^2 + bx + c$, crossing random points, according to the system of equations:

$$\begin{cases} a = \frac{1}{2} y2 + \frac{1}{2} y0 - y1, \\ b = -\frac{1}{2} y0 - t \cdot y2 - t \cdot y0 + 2t \cdot y1 + \frac{1}{2} y2, \\ c = y1 + \frac{1}{2} t^2 \cdot y2 + \frac{1}{2} t^2 \cdot y0 - t^2 \cdot y1 + \frac{1}{2} t \cdot y0 - \frac{1}{2} t \cdot y2. \end{cases} \quad (5)$$

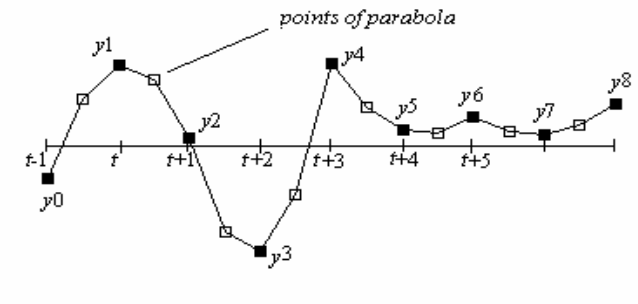


**Fig. 2.** Scheme of the profile simulation by parabolas

These coefficients can be calculated from the following system of equations:

$$\begin{cases} a(t-1)^2 + b(t-1) + c = y0, \\ at^2 + bt + c = y1, \\ a(t+1)^2 + b(t+1) + c = y2. \end{cases} \quad (6)$$

3) selection of higher content of parabola described points in dependence of profile variation;
4) choosing the last point of existing parabola as the first point of parabola describing next profile;
5) generation of two random points that are distributed according to the $\boldsymbol{N}(m, \sigma)$;
6) simulation lasts up to variation $Var$ of calculated points standing to the following rule:

$$\left| Var - \sigma^2 \right| \geq 0.0001$$

Following investigations were performed by connection processes of profiles simulation and abrasion treatment of rubber surface.

## EXPERIMENTAL

For investigation monolithic butadiene-styrene rubber was used. The density and hardness according to Shore scale was $\rho = 0.25$ g/cm$^3$ and $H = 75$, respectively.

To produce rough rubber surface, abrasive paper of different grade number was used. Test pieces in size of $80 \times 25$ mm were abraded on the abrasion machine, which contains special device for applying constant pressing force of $P = 40$ N between specimen and abrasion disk diameter of which was $D = 100$ mm. Abrasion was performed moving sample to abrasion tool at the constant rate of $\nu$=10 mm/s.

The profiles of abrasive and abraded surfaces roughness were registered using digital camera. Analysis of the obtained images converted to the gray scale was performed using a software *ImageJ*.

## RESULTS AND DISCUSSION

In Fig. 3, a, b, views of characteristic digital images of abrasive and abraded rubber surface are presented. In this figure profiles of the surfaces obtained after digital image analysis is presented, also.

After evaluation of the various grade number abrasives and roughened rubber surface profiles, the constants of correlation function Eq. 1 were calculated. The corresponding values are presented in the Table 1.

**Table 1.** Constants of correlation function

| Grade number | Constants of correlation function | | | |
|---|---|---|---|---|
| | Abrasion paper | | Rubber | |
| | $C_{ab} \times 10^{-3}$ | $\alpha_{ab}$ | $C_{rubber} \times 10^{-3}$ | $\alpha_{rubber}$ |
| 24 | 83.8 | 0.163 | 0.17 | 0.243 |
| 40 | 57.4 | 0.244 | 0.29 | 0.423 |
| 60 | 21.3 | 0.252 | 0.49 | 0.449 |
| 80 | 5.8 | 0.232 | 0.50 | 0.464 |
| 100 | 1.4 | 0.200 | 0.52 | 0.508 |

The constants were determined using method of least squares.

After analysis of the profiles statistical parameters it was determined that relation between the coefficients of correlation function and abrasive grade number exist. As the main parameter evaluating the average diameter $G_{av}$ of the grade the abrasion paper grade number was selected.

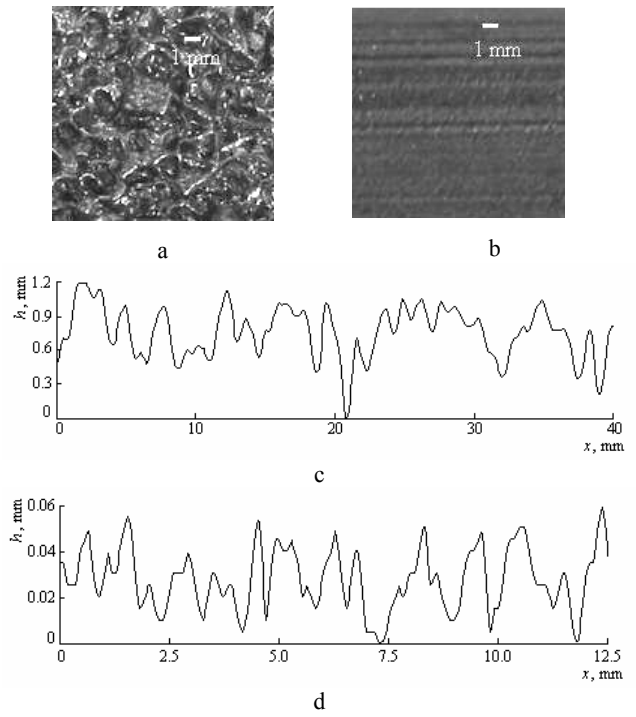


**Fig. 3.** Typical views of surfaces (a) and profile (c) of abrasive N 24 and abraded rubber surface (b) and profile of (d) with abrasive N 24

**Table 2.** Relation between the abrasive number and average diameter of the grade

| Grade number | Average diameter of abrasive grade, mm |
|---|---|
| 24 | 0.545 |
| 40 | 0.388 |
| 60 | 0.269 |
| 80 | 0.201 |
| 100 | 0.162 |

It was determined relation between the coefficients $C_{ab}$, $\alpha_{ab}$ of the abrasive paper correlation function and the grade diameter $G_{av}$:

$$C_{ab}(G_{av}) = 1.52\exp(1.4G_{av}) - 1.55 \ ; \quad (7)$$

$$\alpha_{ab}(G_{av}) = -1.9G_{av}^2 + 1.25G_{av} + 0.05 \ . \quad (8)$$

Correlation coefficient between the experimental results and Eq. 7 and Eq. 8 was equal to 0.99 and 0.98, respectively. Using both of these equations coefficients of the correlation function for various abrasives can be calculated.

Subsequently dependence of the correlation function coefficients of the roughened rubber on the value $G_{av}$ was determined:

$$C_{rubber}(G_{av}) = 1.51(\exp(0.0006G_{av}) - 1) \ ; \quad (9)$$

$$\alpha_{rubber}(G_{av}) = -1.55G_{av}^2 + 0.47G_{av} + 0.45 \ . \quad (10)$$

Correlation coefficients between the experimental data and those calculated according to the Eq. 9 and Eq. 10 are equal to 0.98 and 0.97, respectively.

Eq. 9 and Eq. 10 allows to calculate surface roughness of rubber, abraded with any selected abrasive. So, using Eq. 7 – Eq. 10, coefficients of the correlation function can be calculated without measurements of profile of rubber surface pattern.

As it can be seen, relations presented as Eq. 7 – Eq. 10 depend on the average diameter $G_{av}$ of the abrasive grade. It follows, that absence of these data does not allow to use these equations. Due to that, possibility to calculate coefficients of the correlation functions using relations (i.e. Eq. 3) of spectral density was evaluated as well.

It was determined that expression of process frequencies parameters can be simplified and presented as:

$$|\Phi(i\omega)|^2 = |b_1 i\omega + b_0| = b_1^2\omega^2 + b_0^2 \ , \quad (11)$$

where $\frac{b_0}{a_0} = \frac{m_{rubber}}{m_{ab}}$, $m_{rubber}$, $m_{ab}$ are the average of profiles roughness for rubber and abrasive respectively, it was assumed that $a_0 = 1$. From (11) it follows, that changes of parameter $a_1$ values does not influence results of the spectral density calculations. After this suggestion, using experimental results characteristic process frequency parameters were calculated. Values of them are presented in the Table 3.

**Table 3.** Frequency parameters of the process

| Grade number | $b_1$ | $b_0$ |
|---|---|---|
| 24 | 0.110 | 0.037 |
| 40 | 0.135 | 0.054 |
| 60 | 0.280 | 0.114 |
| 80 | 0.630 | 0.208 |
| 100 | 1.670 | 0.382 |

Later, using data presented in the Table 3 and algorithm presented in the Fig. 1 it is possible to calculate correlation function of roughened rubber surface without its profile and vice versa. To confirm this suggestion the experimental investigations were performed.

Simulation of profiles showed that points of the profiles statistically are distributed according to the normal law. To check this hypothesis criteria of *chi*-square and Kolmogorov-Smirnoff were applied. The constants of the simulated correlation functions and simulated profiles are presented in the Table 4 and in Fig. 4.

As it can be seen, presented the profile simulation method is in good agreement with the experimental results, as the difference between coefficients of correlation function of both experimental and generated profiles is insignificant. Besides, during the simulation additional condition was introduced, which allows that difference between the adjusted and generated dispersion must be not higher than 0.0001.

This condition automatically verifies values of the parameter *C*. Values of generated and experimentally determined parameter $\alpha$ are presented in Fig. 5. There is similar to parameter *C* good agreement of both results. Correlation coefficient for this value is 0.99.

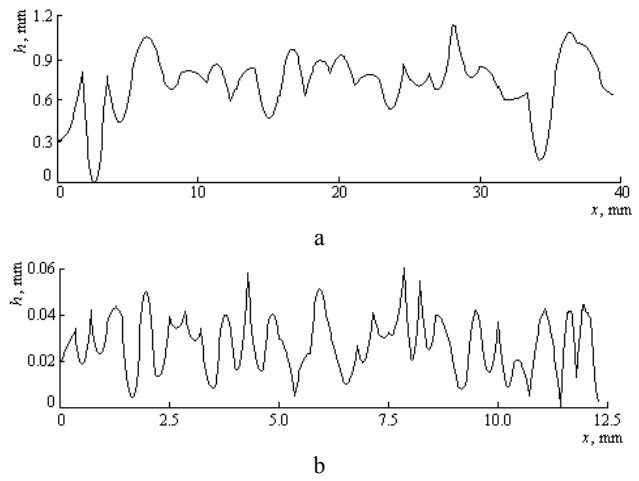


**Fig. 4.** Simulated profiles of abrasive N24 (a) and with it roughened rubber surface profile (b)

**Table 4.** Parameters of the simulated profiles of abrasive and abraded rubber surface

| Grade number | Parameters of correlation function | | | |
|---|---|---|---|---|
| | Abrasive | | Rubber | |
| | $C_{ab} \times 10^{-3}$ | $\alpha_{ab}$ | $C_{rubber} \times 10^{-3}$ | $\alpha_{rubber}$ |
| 24 | 84.2 | 0.165 | 0.18 | 0.242 |
| 40 | 56.9 | 0.247 | 0.30 | 0.425 |
| 60 | 20.7 | 0.250 | 0.48 | 0.447 |
| 80 | 6.0 | 0.233 | 0.49 | 0.468 |
| 100 | 1.5 | 0.210 | 0.51 | 0.512 |

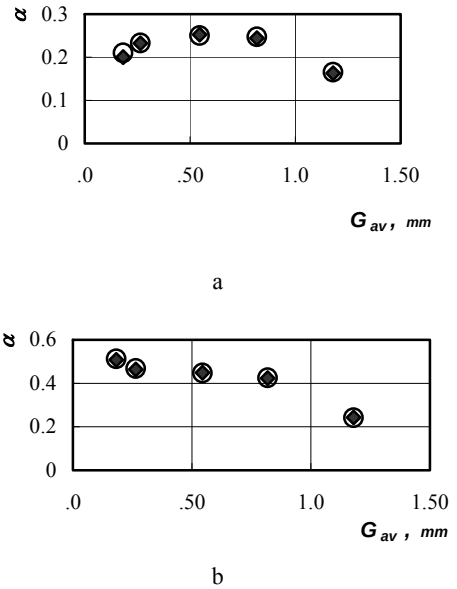


**Fig. 5.** Parameter $\alpha$ of correlation function Eq (1) *vs* diameter of a grade*:* a – for abrasive profile*,* b – for roughened rubber profile: ♦ – experimental points, ○ – generated

For the simulated profiles the algorithm presented in Fig. 1 is also suitable. For example, it is possible to generate profile of the abrasive and using the above presented scheme to generate roughened with appropriate abrasive rubber profile.

## CONCLUSIONS

The simulated profiles coincide with the real profiles. The hypothesis of simulated points distribution according to the normal low $N(m, \sigma)$ was verified by *chi*-square criteria and Kolmogorov-Smirnoff test. Only insignificant difference between the coefficients of correlation function was obtained.

The constructed simulation algorithm fully characterizes abrasion process. It allows to simulate roughened rubber surface profile at selected abrasion conditions (such as abrasive grade number, sample compression force and abrasion velocity). The reversed process is also possible, i.e. when roughened surface profile of rubber is known the grade number of abrasive can be determined. In this case the abrasive tool pressing force and abrasion velocity must be known.

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