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Linear Regression of Relative Elasticity

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The present paper discusses the leather softening by shearing. Under the assumptions that the shearing speed and shearing direction are fixed we establish the procedure which allows us to choose the set of shearing parameters for the softening of a leather sample to 5.8 % relative elasticity. We show that a regime-choosing problem can be treated in an elementary statistical way. The softening regime procedure is based on the linear regressive equation between the relative increase of lether elasticity, shearing angle and the number of shearing cycles. By using statistical methods it is proved that the obtained regressive equation can be used for the relative increase of leather elasticity prediction. The proposed softening procedure is tested experimentally.

Keywords: leather, regression equation, correlation coefficient, residuals, hypothesis.

INTRODUCTION

Research on the development of leather treatment processes is aimed at the development of new leather treatment technologies, the reduction of environment pollution [1 – 3] and stimulation of chrome tanning processes employing pressure [4, 5] as well as focusing on intensified technological processes. In all cases entire pieces of leather are treated.

The authors [6] have experimented with the technology applying which small leather cuttings in a fixed position that were chrome tanned in vacuum. This led to the new processes described in the paper have raised the problem of the development of new leather treatment technologies [6 – 9]. Leather cuttings treated by vacuum and pressure are difficult to be softened by traditional methods. For this purpose leather treatment by shearing was applied. The previous experiments evidenced that the shearing process of leather cuttings results on their softening [9].

The deformation caused by shearing resolves the cohered collagen filaments, thus, alternating the super molecular structure of leather. Consequently, the mechanical characteristics are changed – leather is softened and its elasticity is increased. This is also observed for other forms of mechanical deformation – stretching or bending. The aim of the research is to disclose the possibilities of treatment with chemicals of leather semi-finished of various configurations and alterations of the leather mechanical characteristics during the processes of shearing.

Technology applying which small leather cuttings in a fixed position are chrome tanned by vacuum [10]. Leather cuttings treated by vacuum and pressure are difficult to be softened by traditional methods. For this purpose leather treatment by shearing was applied. The previous experiments evidenced that the shearing process of leather cuttings results on their softening [11].

FORMULATION OF THE PROBLEM

Let us first summarize the discussions in [11, 12]. The authors [12] deduced that the softening effect is a functional dependence of the relative elasticity on the operational factors:

$$E_m = f(y_a, y_v, y_n, y_s, y_k), \% , \tag{1}$$

where y_a is the shearing angle; y_v is the shearing speed; y_n is the number of shearing periods; y_s is the leather initial elasticity; y_k is the sample direction in relations to the leather longitudinal axis. All these factors were divided into two groups: the first three factors in (1) form active factors group, and the leather initial elasticity and the sample direction are passive factors.

Note that the softening effect is definable by the relative increase of leather elasticity, which is the function on leather initial elasticity S_0 and leather ultimate elasticity S_1 :

$$E_m = \frac{S_1 - S_0}{S_0}. \tag{2}$$

Therefore, (1) can be rewritten:

$$E_m = f_1(y_a, y_v, y_n, y_k). \tag{3}$$

The choice of sample direction in relation to the leather longitudinal axis is very problematic in practice. Thus, the factor y_k was eliminated and relative increase of leather elasticity dependences on factors y_a , y_v and y_n [12] were obtained. Dependences are summarized in Fig.1.

Let us fix the choice when the shearing direction goes across the longtitudinal axis of leather and the shearing speed ω = 0.28 rad/s. The aim of this paper is to find good enough approximation of the function

$$E_m = f_2(y_a, y_v), \tag{4}$$

with the goal to use it in creating a leather softening procedure. We will solve this specific problem by using the multiple regression method. In the engeneering and sciences the multiple regression procedures are very widely used in research [13 – 16]. The general purpose of

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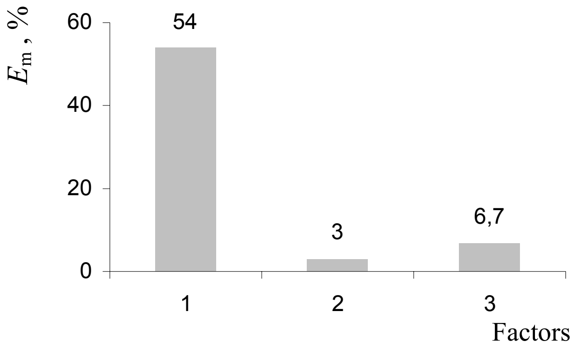

Fig. 1. Dependence of the increase of relative elasticity on the shearing angle (1), the number of shearing periods (2), the shearing speed (3)

the multiple regression is to learn more about the relationship between several independent (predictor) variables and a dependent (criterion) variable.

EXPERIMENTAL

Methods and devices. The leather softening devices are not applicable for the softening small size leather components and for this reason an original device was constructed, which allows to soften pieces of leather by shearing and alternating the angle of shearing, the number of cycles and the speed of the process (Fig. 2).

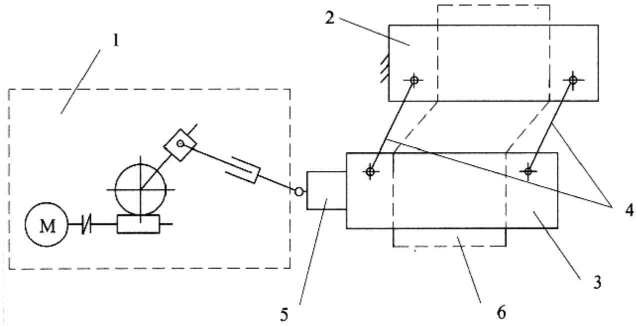

Fig. 2. Scheme device: 1 – gear; 2 – fixed grip; 3 – flexible grip; 4 – levers; 5 – dynamometer; 6 – leather sample

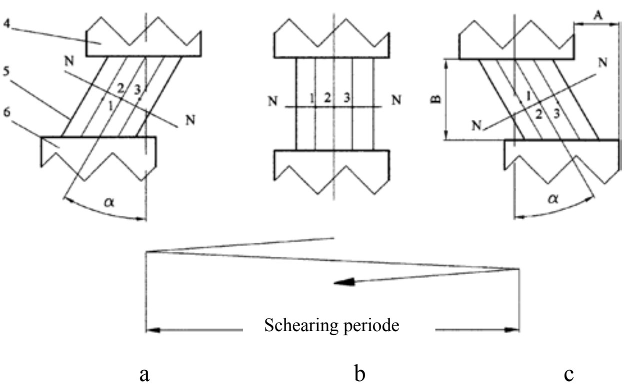

Fig. 3. Scheme deformation of leather sample: 1, 2, 3 – relative points marking collagen filaments; 4 – fixed grip; 5 – leather sample; 6 – flexible grip; A – shift of the grip 6; B – distance between the grips

The basis of leather softening by shearing is the parallel shifting of adjacent collagen filaments into the opposite directions in relation to each other, alternating the direction of shifting during separate shearing cycles (Fig. 3).

The shearing cycle consists of shifting the leather sample 5 between the fixed grip 4 and flexible grip 5 from the initial position (Fig. 3, b) into the left marginal position (Fig. 3, a) at the angle α and its removal into its initial position, followed by shifting it into the right marginal position (Fig. 3, c) at the same angle and its removal to its initial position.

Deformation characteristics of the samples were measured using an electronic dynamometer to measure force and shift. Leather elasticity was measured deforming it by spherical two-spindle stretching [17]. To measure leather elasticity the device was constructed (Fig. 4).

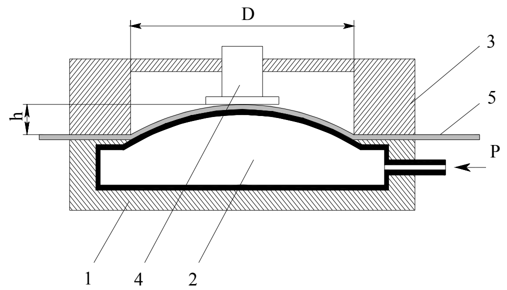

Fig. 4. Pneumatic device occasional to investigate leather mechanical characteristics: 1 – frame; 2 – pneumatic unit; 3 – mandrel; 4 – process sensor; 5 – leather sample

For the first trial 206 leather samples saturated up to the humidity of 20 % – 25 %, in the size of 85 mm × 100 mm, cut from the same non-softened calf leather were taken. The samples were softened by shearing, various combinations of the shearing angle and the number of shearing cycles were used. The experimental data are presented in Fig. 5 and Fig. 6.

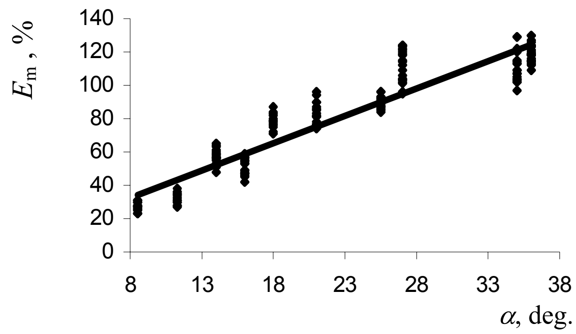

Fig. 5. Dependence of the increase of relative elasticity (E_m) on the shearing angle (α)

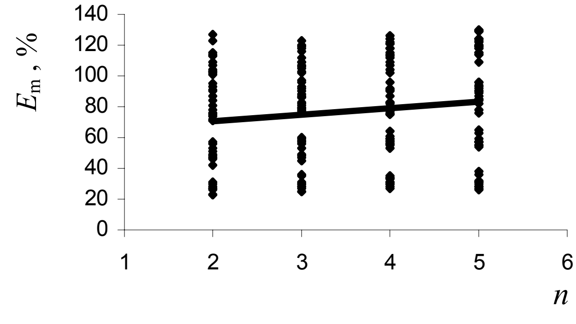

Fig. 6. Dependence of the increase of relative elasticity (E_m) on the number of softening cycles (n)

RESULTS AND DISCUSSIONS

The regression equation. Figure 5 chows a relation (correlation) between the shearing angle and the relative increase of leather elasticity. It can be seen that the relationship between variables is not linear. Therefore, the "linearization" of the shearing angle was used – we considered the natural logarithm of the shearing angle instead of the shearing angle (Fig. 7). Lets put $e_m = E_m / 100$. By using the least square method, we have

$I: \quad e_m = 0.006 + 1.885 y_a,$

$II: \quad e_m = 1.496 + 0.675 \ln(y_a).$

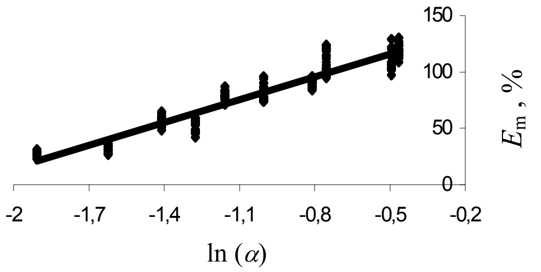

Fig. 7. Dependence of the increase of relative elasticity (E_m) on the natural logarithm of the shearing angle (ln α)

For the both models the squared correlation coefficients (R^2) and standard errors of estimate ($\hat{\sigma}$) were given:

$I: \quad R^2 = 0.884, \hat{\sigma} = 0.110,$

$II: \quad R^2 = 0.919, \hat{\sigma} = 0.009.$

The given results show that the second model explains better (and with a greater precision) the relationship between E_m and the predictor variable.

Fig. 6 summarizes the relationship between y_n and E_m. It can be seen that this relationship is weak, but it is not negligible (see below for details).

The choice of regression model is a complicated issue. We will choose a model that balances among simplicity, theoretical relevance and predictive capability, i.e. consider the multiple linear regression model

$$e_{m,j} = c_0 + c_1 y_{n,j} + c_2 y_j + \varepsilon_j, \tag{5}$$

where $y = \ln(y_a)$; c_0, c_1, c_2 are unknown non-random coefficients; $\varepsilon_j, j = 1,...,205$ are the measuring errors. As usual, let us assume that $\varepsilon_j, j = 1,...,205$ are independent, identically distributed random variables with a zero mean and unknown, finite dispersion.

Table 1. Coefficient estimators of regression (5)

	$k = 0$	$k = 1$	$k = 2$
$\hat{c}_k$	1.393	0.028	0.672
$s(\hat{c}_k)$	0.025	0.005	0.013

By using the least square method the estimators $\hat{c}_0, \hat{c}_1, \hat{c}_2$ of the coefficient c_0, c_1, c_2 were obtained. Table 1 shows the estimators and their standard deviations $s(\hat{c}_0), (\hat{c}_1), s(\hat{c}_2)$.

The values of standard deviations $s(\hat{c}_0)$ and $s(\hat{c}_2)$ do not exceed 2 % of the corresponding estimators values, a relatively high $s(\hat{c}_1)$ value confirms a weak relationship in Fig. 5. High reliability of the estimators (with big weight) indicates good predictive capability.

Thus, the relative increase of the leather elasticity function can be presented on the form:

$$e_m = 1.393 + 0.028 y_n + 0.672 \ln(y_a). \tag{6}$$

Squared multiple correlation coefficient were obtained: $R^2 = 0.929$.

Hypothesis testing. As indicated above (Fig. 1 and Fig. 5), the effect of the number of shearing cycles is low, and a natural question – is the factor y_n a significant predictor – arises. The statistical hypothesis was checked:

null hypothesis: $c_1 = 0,$

alternative hypothesis: $c_1 \neq 0.$

With the significance level $\alpha = 0.05$ the null hypothesis was rejected via the Student test [18]. Thus, the factor y_n can not be ignored in the e_m prediction.

Characteristics of a Relationship. Define the matrix:

$$X = \begin{pmatrix} 1 & 2 & -1.908 \\ 1 & 2 & -1.908 \\ \cdots & \cdots & \cdots \\ 1 & 5 & -0.465 \end{pmatrix},$$

which contains 206 rows. Each element of the first column of the matrix X is a unit, in the next two columns the regimes of softening are represented, e.g. the first nonsoftened sample of leather was softened by using the shearing $y_{n,1} = 2. \ln(y_{a,1}) = -1.908$. It is known [19] that if $\det(X^T X)$ is near zero, then the predictor variables are highly correlated among themselves and it can impede predictive capability. We have

$$\det(X^T X) = 2329540.071.. \tag{7}$$

Given result (7) makes no sign to the predictors VIF test. We have

$VIF(y_n) = VIF(\ln(y_a)) = 1.003.$

Since both the predictors VIF factor do not exceed 4, one can conclude that the relationship between the factors is weak and multicolinearity problem does not exist in the model (6).

Let us define the correlation coefficient between e_m and predictor by $r(e_m, y_n)$. Where obtained values are

$r(e_m, y_n) = -0.051, r(e_m, \ln(y_a)) = 0.959,$

which revealed that the increase of relative elasticity is strongly related to $\ln(y_a)$.

Residuals analysis. Note that the residual is the difference between the predicted value:

$$\hat{\varepsilon}_j = e_{m,j} - 1.393 - 0.028 y_{n,j} + 0.672 y_j. \tag{8}$$

The residuals (8) histogram is presented in Fig. 8.

By using (8) the average and standard deviation the residuals are calculated:

$\bar{\hat{\varepsilon}} = 0, \hat{\sigma} = 0.087.$

Let us test the hypothesis:

null hypothesis: residuals ε_j have Gaussian distribution (see its probability density function in Fig. 6),

alternative hypothesis: residuals ε_j are not distributed normally.

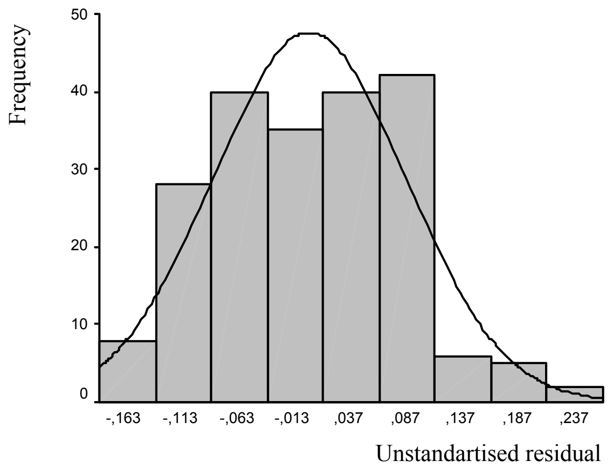

Fig. 8. The residual histogram

Let us choose the significance level $\alpha = 0.05$. The Chi-square Test [19, 20] was used. Calculations show that we must reject H_0. Since the residuals histogram is not very asymmetric, for the practical purposes it is convenient to assume that the residuals are normally distributed, and to predict the e_m at a given value of the shearing angle y_a simply by solving the equation

$$\hat{e}_m = 1.449 + 0.672 \ln(y_a). \tag{9}$$

Here we fixed the number of shearing cycles at $y_n = 2$ with the goal to choose an optimal shearing regime. With a 95 % probability the absolute value of the difference between the true value of e_m and predicted value $\hat{e}_m$ do not exceed

$$0{,}176\sqrt{1.043 + 0.054 \ln(y_a) + 0.023(\ln(y_a))^3}\ . \tag{10}$$

Parameter M. Let us note that equation (9) is "personalized" – it was obtained by softening the same calf leather samples. Thus, equation (9) is corrected by introducing parameter M :

$$\hat{e}_m = 1.449 + 0.672 \ln(y_a) + M, \tag{11}$$

The value of parameter M is individual for each calf and it shows how well the given calf leather softens (by shearing) with regard to the examined calf leather.

PROCEDURE FOR LEATHER SOFTENING BY SHEARING

The next two-step procedure selects the softening regime (to the ultimate elasticity 5.8 %):

1. Parameter M identification.

a) Choose random N samples (we recommend $5 \le N \le 7$) and detect their initial elasticity $\Delta S_{0,1}, \Delta S_{0,2}, \ldots, \Delta S_{0,N}$.

b) Soften the samples by shearing and detect their ultimate elasticity $\Delta S_{1,1}, \Delta S_{1,2}, \ldots, \Delta S_{1,N}$.

Use shearing parameters:

$y_{a,k} = \gamma, y_{n,k} = 2, k = 1, \ldots, N.$

We recommend $\gamma = 20^\circ$.

c) By using (2) calculate increases of the relative elasticity $E_{m,1}, E_{m,2}, \ldots, E_{m,N}$ and find the average value $\overline{E}_m$.

d) Find the value of parameter M from

$M = 0.01\overline{E}_m - 1.449 - 0.672 \ln(\gamma).$

2. Choosing the leather-softening regime.

e) For the chosen sample detect its initial elasticity S_0. By using (2) and S_1=5.8 % find a proper increase of the relative elasticity E_m.

f) Find y_a by solving equation (11).

g) Use shearing parameters $y_n = 2$ and y_a for the sample softening.

Test. 78 leather samples were used for testing the softening procedure. The test parameters: $N = 7$, $\gamma = 20^\circ$. The experimental results allowed finding $\overline{E}_m = 44.6$ % and $M = -0.296$. In view of (9), the softening angle (for each sample separately) was founded from

$$y_a = \exp\left(\frac{10E_m - 1153}{672}\right),\ \text{[rad.]} \tag{12}$$

The test results are presented in Fig. 9. The upper and the lower curve equations are:

$$y_{\pm} = 115.3 + 67.2 \ln(y_a) \pm 100D(y_a), \tag{13}$$

where $D(y_a)$ is defined in (10).

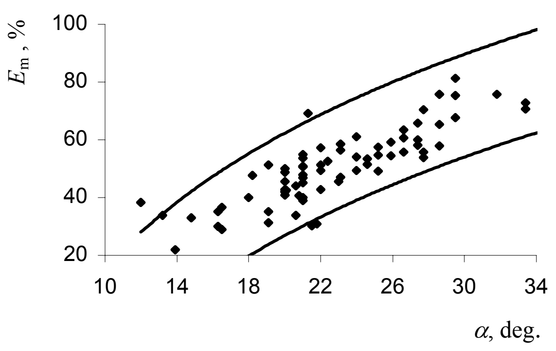

Fig. 9. The test results

The quality of the obtained regression equation (11) and suggested leather shearing procedure defines the test results. Seven samples from 71, used in the test, were used in parameter M identification. Four samples from the rest 63 samples (i.e. 6.3 %) do not come into the functional segment.

$$1.153 + 0.672 \ln(y_a) \pm D(y_a). \tag{14}$$

Why is the outcome not 95 % of successfully sheared samples? The answer is in parameter M. In fact, it is a random variable. Thus, parameter M identification brings in new errors. The research of its probabilistic distribution is practically complicated. On the other hand, 93.7 % of successful tests show that 7 samples are enough to detect parameter M .

CONCLUSIONS

A regressive function of relative elasticity ideally coressponding to the data of the experiment has been worked out. Using it according to the initial and projected elasticity values of leather, it is possible to accurately predict the parameters of softening by shearing operating conditions.

The recommended algorithm and constructed device for softening the whole piece of leather could serve as theoretical basis for further research on leather softening by shearing.

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