

## Simulation and Experimental Research of the Process of Impulse Densification of Moulding Sands

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Received 31 May 2005; accepted 07 July 2005

The paper presents a mathematical model that completely describes the process of impulse densification of moulding sands. The developed model consists of a set of differential equations describing the impulse head dynamics and the process of deformation and densification of moulding sands. Mathematical modelling of the dynamic squeezing process was based on the rheological model of moulding sand developed on the basis of its rheological properties identified experimentally by the time characteristics method. The developed model can be used in simulation research for selecting and optimising the parameters of impulse moulding machines, as well as for optimising the process of impulse densification of moulding sands.

*Keywords:* impulse head, moulding sands, rheological model, simulation research, experimental research.

### 1. INTRODUCTION

Recently, a significant development of dynamic versions of squeezing, i.e. impulse densification and dynamic pressing have been occurred, resulting in their currently almost exclusive use for dynamic densification of traditional moulding sands.

To obtain optimum results of moulding sand impulse densification, it is necessary to know the mathematical model of the densification process and the results of that model simulation research. The mathematical model should include description of the impulse head dynamics and the process of the sand deformation and densification.

Because of complexity of the phenomena occurring during dynamic densification of moulding sands, any mathematical model fully describing the process has not been so far developed, although such trials were undertaken by many scientists all over the world, among others G. M. Orlov [1], J. Bast [2] and K. Smyksy [3]. The so far developed models do not include any complex description of the impulse densification process, and thus they can not be used either for simulation research or for designing and optimising the process.

A trial to develop a mathematical model fully describing the process of the moulding sand densification process was undertaken [4, 5]. The model was based on description of the impulse head dynamics and the deformation and densification process.

The developed model can be used, among others, for:

- selecting design parameters and operating conditions of impulse machine heads,
- selecting parameters of the impulse densification process,
- optimising moulding machine heads and the densification process.

The goal of this paper is to present the so-far unsolved problem of mathematical modelling of the impulse

densification process of moulding sands. The introduced model can be used for simulation research of the impulse process. Then, results of a simulation research can make a ground for designing and optimising impulse heads of moulding machines and the process of moulding sand dynamic densification.

### 2. MATHEMATICAL MODEL OF IMPULSE DENSIFICATION OF MOULDING SANDS

Modelling of impulse densification of moulding sands requires joint considering the mathematical models of:

- impulse head,
- deformation and densification process.

The base for development of the deformation and densification process is a rheological model of moulding sand developed by the authors using the time method [6]. The model was determined on the basis of analysis of a stepwise characteristics  $h(t)$  that presents a time-relationship of unit pressures in moulding sand caused by a stepwise load.

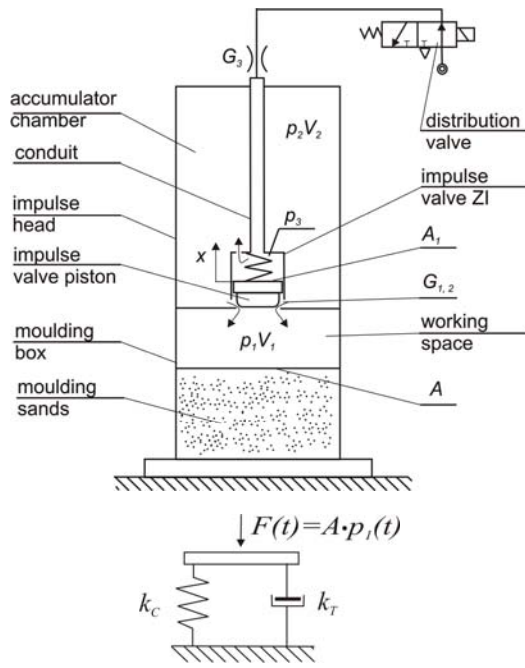
It was found that response of the densification sand to a stepwise input function is a strongly suppressed decaying oscillation, i.e. it behaves as an oscillating component. A viscoelastic rheological model was accepted to describe rheological properties of the moulding sand.

Diagram of the impulse densification process is shown in Fig. 1.

Considering the impulse head dynamic model and the sand rheological model, the impulse densification process can be described by a set of differential equations, considering some simplifying pre-assumptions:

- air is an ideal gas,
- thermodynamic processes are of quasi-static nature,
- no heat exchange occurs between the air in the working space and the environment,
- air temperature change during the impulse process is negligible and frictional resistance in seals is negligible.

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**Fig. 1.** Diagram of the impulse densification process of moulding sands

$$p_1 \cdot A_1 = m_2 \cdot \frac{d^2 y(t)}{dt^2} + k_T(\delta) \cdot \frac{dy(t)}{dt} + k_C(\delta) \cdot y(t), \quad (1)$$

$$A_1(p_2 - p_3) - c \cdot (x + y_0) - m_1 \cdot g = m_1 \cdot \frac{d^2 x}{dt^2}, \quad (2)$$

$$\frac{-\kappa \cdot R \cdot T \cdot G_2}{V_2} = \frac{dp_2}{dt}, \quad (3)$$

$$\frac{\kappa}{s-x} \cdot \left( p_3 \cdot \frac{dx}{dt} - \frac{G_3 \cdot R \cdot T}{A_1} \right) = \frac{dp_3}{dt}, \quad (4)$$

$$\frac{\kappa}{y} \cdot \left( \frac{G_1 \cdot R \cdot T}{A_1} - p_1 \cdot \frac{dy}{dt} \right) = \frac{dp_1}{dt}, \quad (5)$$

where  $m_1$  is the mass of movable parts of the impulse valve,  $m_2$  is the mass of a portion of the moulding sand to be densified,  $x$  is the co-ordinate of the valve piston ZI,  $y_0$  is the initial (assembly) deflection of the valve spring,  $c$  is the spring rate,  $p_1$ ,  $p_2$ ,  $p_3$  are the absolute pressure, respectively: in the accumulator chamber, in the working space and in the impulse valve return chamber,  $A_1$  is the cross-section area of the piston,  $s$  is the stroke of the piston,  $g$  is the gravitational acceleration,  $\kappa$  is the adiabate exponent,  $G_i$  is the air outflow rate from the  $i$ -th chamber,  $R$  is the gas constant,  $T$  is the temperature of the air in the working space and in the accumulator,  $V_1$  is the volume of the working space,  $V_2$  is the volume of the accumulator.

Individual equations (1) – (5) describe:

- equation (1) – deformation process of moulding sand,
- equation (2) – movement of the valve piston ZI,
- equations (3) – (5) – gas transformations, respectively in the accumulator chamber, in the impulse valve working chamber and in the working space above the moulding sand.

During the densification process, the sand is exposed to deformation with no side distortion that is limited by the

moulding box. This results in plastic flow and consolidation (densification) of the moulding sand.

Analysis of the deformation and densification process leads to the following conclusions:

- in non-stationary states, total pressure in the moulding sand represents a sum of the pressures transmitted by the viscous and the elastic elements and the pressure resulting from the sand densification (value dependent on densification degree);
- in stationary state after dynamic squeezing, total pressure in the moulding sand represents a sum of the pressures resulting in the pressing force and the sand densification that brings it to a determined strength.

Therefore, total pressure in the dynamically squeezed moulding sand can be described by the relationship:

$$p_C = \frac{k_C(\delta) \cdot x + k_T(\delta) \cdot \dot{x}}{A} + p_U(\delta), \quad (6)$$

where  $p_C$  is the total pressure in the moulding sand,  $p_U$  is the pressure resulting from the sand densification,  $A$  is the cross-section area of the moulding box.

Consolidation (densification) of the moulding sand can be described by the experimental relationship:

$$p_U = k(p_0) \cdot E(\delta) \cdot \text{tg} \delta, \quad (7)$$

$$k(p_0) = d_1 \cdot p_0^2 + d_2 \cdot p_0 + d_3,$$

where  $d_i$  are the coefficients,  $E = f(\delta)$  is the elasticity modulus in function of the densification degree.

The modulus of elasticity  $E(\delta)$  can be calculated from the relationship [7, 8]:

$$v_L(\delta) = \sqrt{\frac{E(\delta)}{\rho(\delta)}}, \quad (8)$$

where  $v_L(\delta)$  is the velocity of ultrasonic longitudinal wave in the moulding sand in function of its densification degree,  $\rho(\delta)$  is the apparent density of the moulding sand in function of its densification degree.

After a transformation of the relationship (8), the elasticity modulus can be expressed as:

$$E(\delta) = v_L^2(\delta) \cdot \rho(\delta). \quad (9)$$

To carry out a simulation research of the mathematical model of the impulse densification process, it is necessary to know the relationships  $k_T(\delta)$  and  $k_C(\delta)$  that respectively determine changes of the sand viscous and elastic properties during its dynamic squeezing. The relationships  $k_T(\delta)$  and  $k_C(\delta)$  can be determined on the basis of experimental research consisting in velocity  $v_L$  measurements of ultrasonic longitudinal wave in function of the moulding sand densification degree.

Changes of viscous and elastic properties of the moulding sand are determined by the relationships:

$$k_T(\delta) = a_1 \cdot \exp[a_2 \cdot v_L(\delta)], \quad (10)$$

$$k_C(\delta) = b_1 \cdot \exp[b_2 \cdot v_L(\delta)], \quad (11)$$

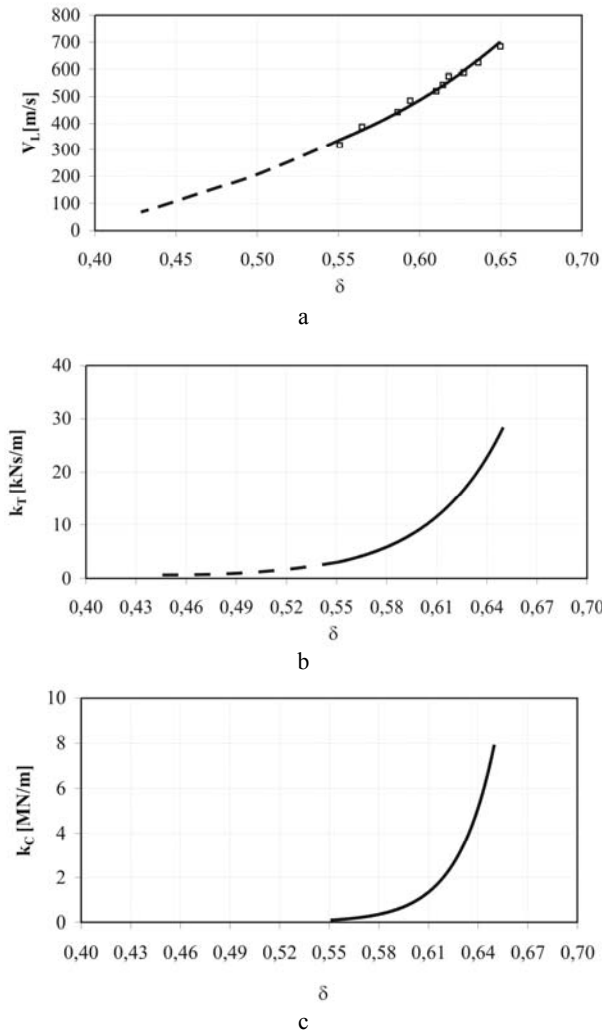
where  $a_i$ ,  $b_i$  are the coefficients.

On the basis of analysis of the above-presented mathematical model, it can be found that ultrasonic examination of the moulding sand makes the necessary and sufficient condition for a simulation research of the

impulse densification process. Knowledge of the coefficients  $k_T(\delta)$  (10) and  $k_C(\delta)$  (11) that determine viscous and elastic properties of moulding sand in function of its densification degree makes a basis for application of the developed model in simulation research of the moulding sand densification process.

### 3. RESULTS OF THE RESEARCH

Simulation research of the developed mathematical model and experimental research of the process of dynamic squeezing of moulding sands were carried out in the following phases:

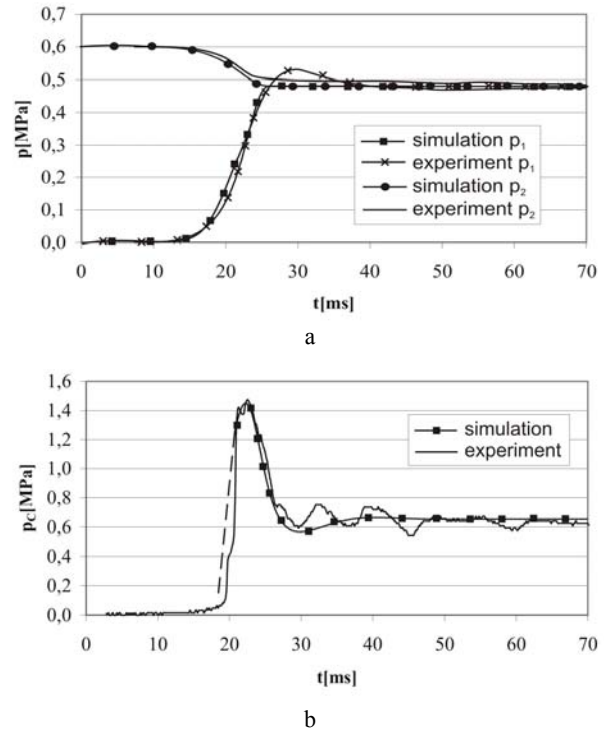


**Fig. 2.** Measurements of ultrasonic wave velocity  $v_L(\delta)$ ; a) and the relationships  $k_T=f(\delta)$ ; b) and  $k_C=f(\delta)$ ; c) for a moulding sand with 6 % of Geco bentonite and humidity  $W = 2.40\%$

- first, ultrasonic examination of the moulding sand was carried out to determine the relationship  $v_L=f(\delta)$ ,
- next, approximation was performed of the relationships  $k_C=f(\delta)$  and  $k_T=f(\delta)$  that characterise parameters of the rheological model of the moulding sand, i.e. its viscous  $k_T=f(\delta)$  and elastic  $k_C=f(\delta)$  properties,
- then, the experimentally determined relationships characterising rheological properties of the moulding sand were applied in simulation research of the developed mathematical model,

- then, experimental research of the impulse densification process was carried out,
- finally, the results of simulation and experimental research were analysed.

Fig. 2 shows the results of ultrasonic measurements [ $v_L=f(\delta)$ ] and the relationships  $k_C=f(\delta)$  and  $k_T=f(\delta)$  for the moulding sand with 6 % of Geco bentonite and humidity  $W = 2.40\%$ .



**Fig. 3.** Results of simulation and experimental research of the impulse densification process of a moulding sand with 6 % of Geco bentonite and humidity  $W = 2.40\%$ : changes of pressures in the working space ( $p_1$ ) and in the accumulator tank ( $p_2$ ) (a), total pressure  $p_C$  in the moulding sand (b) at the initial supply pressure  $p_0 = 0.60$  MPa

In Fig. 3, the simulation and experimental results of impulse densification of the considered moulding sand are presented. Fig. 3, a, shows changes of pressure in the working space ( $p_1$ ) and in the accumulator tank ( $p_2$ ), and Fig. 3, b, shows total pressure  $p_C$  in the moulding sand at the initial supply pressure  $p_0 = 0.60$  MPa.

The obtained results prove that the developed mathematical model of the impulse densification process describes its dynamics with a practically very good approximation, both qualitatively and quantitatively. In addition, it should be emphasised that this is the first model that describes the impulse densification process completely and makes simulation research of this process possible.

### 4. CONCLUSIONS

The mathematical model of the impulse densification process of moulding sands was based on:

- the model of the impulse head dynamics,
- the model of deformation and densification of moulding sands, developed on the ground of a rheological model of the moulding sand that was experimentally identified by the time-characteristics method.

The rheological model of the moulding sand was determined by analysis of the stepwise characteristic  $h(t)$  representing time function of pressures  $p_C$  in the moulding sand, resulting from the stepwise pressing load. Analysis of the  $h(t)$  characteristic lead to the conclusion that response of the moulding sand to a stepwise input function has the nature of a very strongly damped oscillation. On this basis, the viscoelastic rheological model was accepted for description of the moulding sand rheological properties.

The presented results of simulation research of the process of impulse densification of moulding sands justify the following conclusions:

- rheological properties of a moulding sand can be modelled by a viscoelastic rheological model.
- the relationships  $k_C=f(\delta)$  and  $k_T=f(\delta)$ , describing elastic and viscous properties of the moulding sand, can be determined on the basis of ultrasonic examination of the sand.
- knowledge of the relationship  $v_L=f(\delta)$  is the only precondition to determine the relationships  $k_C=f(\delta)$  and  $k_T=f(\delta)$  and to carry out simulation research of the developed mathematical model of the impulse densification process.
- the course of the impulse densification process can be evaluated on the ground of the relationship  $p_C=f(t)$ . This relationship can be used for evaluating strength of a dynamically densified moulding sand. The pressure  $p_U$  is equivalent to the strength of the impulse densified moulding sand.  
The developed mathematical model can be used for:
- selecting design parameters and operating conditions of impulse heads,
- selecting parameters of impulse densification of moulding sands,
- optimising design parameters of moulding machine heads,
- optimising process parameters of impulse densification of moulding sands.

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