ISSN 1392–1320 MATERIALS SCIENCE (MEDŽIAGOTYRA). Vol. 9, No. 4. 2003

Hypothetical Models of Yarn Cross-section: Comparison of the Shapes and Packing Peculiarities

D. Petrulis*, **S. Petrulytė**

Department of Textile Technology, Kaunas University of Technology, Studentų 56, LT-3031 Kaunas, Lithuania

Received 30 September 2003; accepted 20 October 2003

In this paper the models of morphological structure with respect to shape and packing properties of yarn cross-section are analysed. Classification of the modes of yarn cross-section shapes is proposed. According to this classification four types of yarn cross-section shapes, i.e., basic, derivative, combinative, and compound are distinguished. The possibility to compare different models is discussed also the main assumptions on this point are suggested. The hypothetical cross-sectional models, namely open packing, quadratic packing, solid packing, close packing, etc. are compared. Some new suggestions are made for the comparison of the close-packed and open-packed yarn models.

Keywords: cross-sectional model, yarn cross-section, yarn packing, yarn structure.

INTRODUCTION

The models of yarn cross-section as an object of structural morphology have been extensively studied over many years.

According to various experimental data, the structure of real yarns is quite disordered, and is replaced by greatly idealised structure of yarn models. This can lead to a hypothetical yarn cross-sectional structure with properties that are rather far from reality. Therefore various types of yarn packing models can exist.

Various properties of yarn cross-section, for instance, such subjects as the modes of cross-sectional shapes, the packing indices of these models are investigated in [1 – 16]. In most cases the open-packed and the close-packed hypothetical models are used [1]. As a rule, one of these main structural models of yarn packing is chosen, but other types of structure, for example, quadratic packing [4], solid packing [5] or different types of hexagonal packing [6] are possible, too. For example, the concept of a gap between filaments in cross-section of a yarn was introduced for hexagonal model in [6]. Therefore this hexagonal model can have not only a well known limited closed structure but other forms are available, too, i.e., a loose packing, a transitional structure or a compact arrangement of the filaments. A major simplification of solid modelling, which was suggested by Keefe [5] is that the shape of the strand cross-section is perfectly elliptical. According to Grishanov et al. [7] this produces a model in which the space within a yarn that is unoccupied by fibres is much greater than that which occurs in the real yarns. A model suggested by Grishanov et al. [7] that takes account of the compression deformation of the individual strands of two-component yarn caused by transverse forces generated during the twisting process. Although the results of cross-sectional packing were not presented in that paper, the new approach is hopeful as a simulated cross-section has proved to consistent with experimental data.

An idealisation of yarn cross-section may be used for prognosis of various properties of yarns [4, 6, 8 – 10] such like yarn diameter, twist, contraction, mechanical indices. That is why an important task is to compare various structural properties, including the shape of yarn cross-section, the position of filaments (fibres) and the packing peculiarities of above-mentioned models.

The current paper is devoted to the comparison of structural properties of these models. An agreement between the models and experimental data requires a detailed consideration that is beyond the aim of this research.

RESULTS AND DISCUSSIONS

Modes of Yarn Cross-section Shapes

A variety of cross-sectional shapes of yarn structural models can be explained by the classification presented in Fig. 1. These shapes differ in complexity. A view of the cross-sectional shapes is given in the corresponding references [1 – 3, 5, 6, 10, 11, 14].

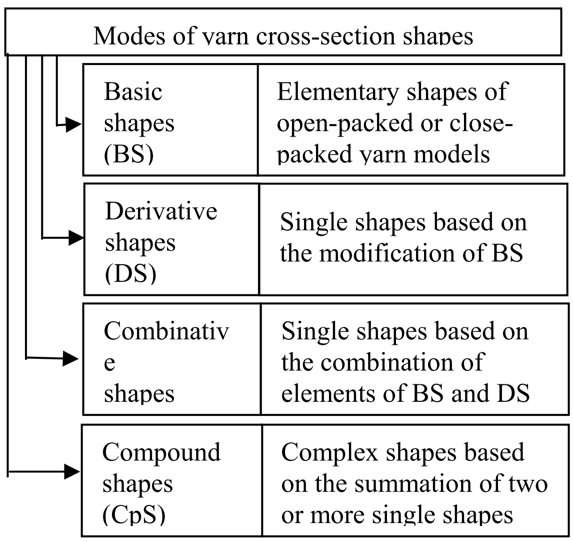

Fig. 1. Classification of modes of yarn cross-section shapes

*Coresponding author. Tel.: + 370-37-353862.; fax: + 370-37-353989.
E-mail address: *Donatas.Petrulis@.ktu.lt* (D. Petrulis)

A round shape and an equilateral hexagonal shape are the simplest modes named as basic shapes (BS). These models are used most of all because their simplicity.

Second type of modes, which are described in proposed classification as derivative shapes (DS), can be defined as modifications of above-mentioned BS. For instance, the round shape may be modified into elliptical shape or into the shape of lens. Various irregular hexagonal shapes are modifications of equilateral hexagonal shape.

Finally, more complex combinative and compound shapes (CbS and CpS, respectively) are possible.

It is worth to note that in the research of woven fabrics geometry the DS and CbS of a yarn cross-section are often far from round. A case of the CpS may be applied to the twisted yarns.

Main Assumptions of Comparison

The possibility to compare packing models depends on the conditions related with mathematical description of each model. Moreover, all assumptions of the investigation must be in unity for these models. For instance, as a basis for all analysed models an assumption that the yarn is composed of equal, non-compressible, circular filaments is held. Another fundamental assumption introduced is that during twisting, the filaments are inextensible. One more assumption is related to a shape of yarn cross-section. In order to receive comparable results for each model a yarn is assumed to have fixed shape of cross-section. Besides that, a diameter of each yarn model remains in a fixed level if yarn twist is constant.

The open-packed and the close-packed models of yarn cross-section differ not only in total number of filaments in the yarn of the same diameter (index n) but also in other indices, for example, in the number of cross-sectional layers t, in quantity of packing fraction Φ, etc. It should be observed that the cross-section of open-packed model is round. Meanwhile in the case of close packing the filaments fit into a hexagonal pattern. However, the possibility of comparison may exist if the same shape of yarn cross-section is selected. The experimental material, which was accumulated by the other investigators, confirms that a round cross-section of the yarn is more acceptable. So, identical approximation to circle was used to compare both structural models. The method of such theoretical solution was suggested in [2].

Peculiarities of Packing Indices

One of the indices, which is used in the current investigation, is Δn. This index indicates a difference of filaments in a cross-section of a yarn if the filaments are arranged according to close-packed model and open packed model. A condition of equal yarn diameter was used in this comparison. The quantities of number of filaments indicated in [2, 3] were used. The computing was carried out for a number of ring layers t varying from 1 to 12. The results of the difference Δn are illustrated in Fig. 2. Through the identity of both the models in the case of $t \leq 3$ index $\Delta n = 0$. The round cross-sectional structure of close-packed model and open-packed model differ in value n if the $t > 3$. It was established that in a range of t from 4 to 10 inclusively, the difference Δn grows according linear equation:

$$\Delta n = 6(t-3) \quad . \tag{1}$$

Further sharp increase of the difference Δn was received for $t = 11$ and $t = 12$ because of considerable growth in the number of filaments [2, 10] for close-packed structure. Moreover, starting at the $t = 11$ the difference Δn reaches a quantity above the number of filaments in corresponding layer of open-packed structure. For example, for the $t = 11$ the difference Δn is equal to 66 as it can be seen in Fig. 2. Meanwhile, the number of filaments in eleventh layer of open-packed yarn is 60. Therefore, when the number of filaments in the yarn is large and constant, both models of yarn cross-sectional packing differ in a number of layers t, too. For example, there are 11 layers in the cross-section of yarn with 397 close-packed filaments [2]. In contrast, there are 12 layers in the cross-section if these filaments are arranged according to open–packed model [11].

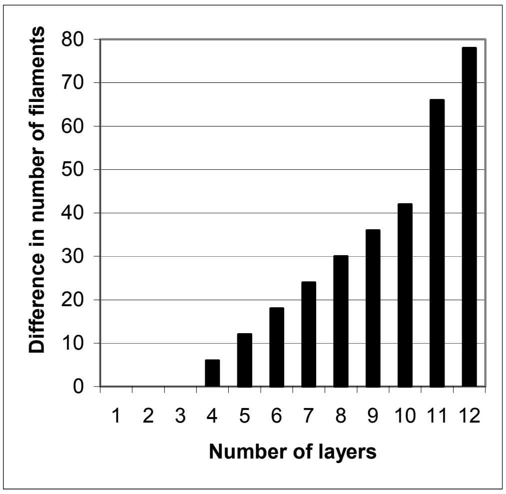

Fig. 2. Difference in the number of filaments (index Δn) against the number of layers t in yarn cross-section

The other index, which was used for the estimation of yarn cross-section packing magnitude, is the packing fraction Φ. Data of theoretical values of Φ for different packing models is presented in the Table 1. These values are arranged by increasing order. The results of packing fraction of most models are given to single yarns, but rather often the same model is used for more complex structures or components: plied yarns, ropes, components of various fancy and covered yarns, etc. An exclusive is a solid model suggested by Keefe [5]. This model is applied to rope-like structure composed of a twisted bundle of individual yarns.

The results show that firstly, the packing fraction Φ depends on packing model. Secondly, only a monofilament yarn has the maximum packing fraction, while for the other types of yarns the packing fraction is less. The smallest packing fraction exists for loose, transitional, and open packing models. The packing fraction is the greatest

Table 1. The theoretical values of indices of yarn packing fraction Φ according to literary data

Packing model	Number of filaments n	Number of cross-sectional layers t	Packing fraction Φ	Source of information
Loose	–	–	Up to 0.2	Neckar, Jezek [6]
Transitional	–	–	0.2 – 0.4	Neckar, Jezek [6]
Compact	–	–	0.4 – 0.9	Neckar, Jezek [6]
Open	Up to 173	Up to 8	0.775 (average value)	Binkevičius [9]
Open	Up to 397	Up to 12	0.750 – 1.000	Petrulis [11]
Open	Up to 217	2 – 9	0.751 – 0.778	Hearle, Merchant [12]
Quadratic	–	–	0.785	Perepelkin [4]
Solid	Case of multi-yarn	–	0.785	Keefe [5]
For 3-D woven materials	–	–	0.8	Miravete [17]
Close	Up to 475	Up to 12	0.760 – 1.000	Petrulis, Petrulytė [2, 3]
Close	475	12	0.898	Petrulis, Petrulytė [3]
Close	–	–	0.9	Morris, Merkin, Rennell [13]
Close	–	–	0.906	Hearle, Merchant [12]
Close	Unlimited large	Unlimited large	0.906	Petrulis, Petrulytė [2]
Close	Unlimited large	–	0.9069	Iyer Balakrishna, Phatarfod [14]
Close	–	–	0.907	Perepelkin [4]
Close	–	–	0.907	Neckar, Jezek [6]
Close	Unlimited large	–	0.9078	Gracie [15]
Close	–	–	0.91	Hearle [1]
Close	–	–	0.92	Hearle [16]
–	1	1	1.000	Neckar, Jezek [6]

quantitatively if the filaments are arranged in yarn cross-section according to close-packed model. Intermediate values of index Φ were obtained if quadratic or solid packing model was chosen. Rather wide range of packing fraction is presented for compact model.

The influence of such indices as the number of filaments in a yarn n and the number of layers in a yarn cross-section t is important for the quantity of packing fraction, too. In [2, 3] it was shown that for low numbers of n or for corresponding number of t the packing fraction fluctuates. With the further increase of n or t the packing fraction quantity is almost constant. Moreover, for unlimitedly large number of n or t the packing fraction Φ reaches the limiting value.

New Suggestions

In this study the packing fraction Φ of two main models (close and open packing) was compared by means of the ratio of their quantities c.

As in previous case, the investigation was carried out for a number of cross-sectional layers ranging from 1 to 12. The ratio c is non-other as the ratio of number of filaments in yarns located according to close-packed and open-packed models.

As can be seen from the Table 2 the minimal value of ratio c was obtained in the case when the number of layers $t \leq 3$. In the range of index t from 4 up to 12 the quantity of ratio c is rather stable. It fluctuates between 1.083 and 1.198.

Table 2. The results of ratio c computations

Number of cross-sectional layers t	Number of filaments n		Ratio c
	For close-packed model	For open-packed model	
1	1	1	1.000
2	7	7	1.000
3	19	19	1.000
4	43	37	1.162
5	73	61	1.197
6	109	91	1.198
7	151	127	1.189
8	199	169	1.178
9	253	217	1.083
10	313	271	1.155
11	397	331	1.199
12	475	397	1.196

The mean value of the ratio c for the whole interval of this research is 1.130. In other words, the values of Φ for close packing and open packing show the difference about 13.0 % if the index t varies between 1 and 12.

In addition the limiting value of ratio c was estimated. An investigation was carried out for the number of layers t which increases to unlimited large quantity:

$$c = \lim_{t \to +\infty} \frac{\pi\left(4t^2 - 4t + 1\right)}{2\sqrt{3}\left(3t^2 - 3t + 1\right)} = \\ = \frac{\pi}{2\sqrt{3}} \lim_{t \to +\infty} \frac{4 - (4/t) + \left(1/t^2\right)}{3 - (3/t) + \left(1/t^2\right)} = 1.209 . \quad (2)$$

It is possible to get the same result if the number of filaments in the yarn n increases:

$$c = \lim_{n \to +\infty} \frac{\pi(4n-1)}{6\sqrt{3}n} = \frac{\pi}{6\sqrt{3}} \lim_{n \to +\infty} \left(4 - \frac{1}{n}\right) = 1.209 . \quad (3)$$

These results show that for the same diameter of yarn the greatest difference in packing fraction (approximately 21 %) exists if the number of cross-sectional layers t or the number of filaments n in the yarn infinitely increases. Approximately the same maximum of this difference can be obtained directly if the limiting values of the indices Φ for close-packed and open-packed models presented in Table 1 are compared: $c = 0.906 : 0.750 = 1.208$.

CONCLUSIONS

According to the proposed classification of shape modes four types of yarn cross-section shape, i.e., basic, derivative, combinative, and compound shapes were distinguished. In order to compare packing fractions for different models a ratio of their quantities can be introduced. By theoretical investigation of close-packed model and open-packed model, when the number of filaments or the number of cross-sectional layers is infinitely large, it was showed that the limiting difference in packing fraction is 21 %. One more result of comparison lies in the difference of the number of filaments Δn for these models. When the range of number of ring layers t is rather wide (t fluctuates between 4 and 10) the index Δn is connected with the index t by means of linear equation.

REFERENCES

1. **Hearle, J. W. S., Grosberg, P., Backer, S.** Structural Mechanics of Fibres, Yarns, and Fabrics. Wiley-Interscience, 1969.
2. **Petrulis, D., Petrulytė, S.** Properties of Close Packing of Filaments in Yarn *Fibres & Textiles in Eastern Europe* 40 (1) 2003: pp. 16 – 20.
3. **Petrulis, D., Petrulytė, S.** Mathematical Description of Filament Position in Close-packed Yarn Cross-section and the Characteristics of Yarn Close Packing *Materials Science* (*Medžiagotyra*) 2 (8) 2002: pp. 202 – 204.
4. **Perepelkin, K. E.** 4 (Four) – Level Textiles Structure. Physical Fundamentals and Methodology of Properties Prognosis *Materials of 'Conference Arch Tex' 97 Textiles Designing & Care'*, Lodz, Poland, 1997: pp. 247 – 255.
5. **Keefe, M.** Solid Modelling Applied to Fibrous Assemblies. Part I: Twisted Yarns *J. Text. Inst.* 85 1994: pp. 338 – 349.
6. **Neckar, B., Jezek, H.** Influence Exerted by the Spinning System and the Fibre Properties on Staple Fibre Yarns *Melliand Textilber.* 7 1985: pp. 481 – 485 (in German).
7. **Grishanov, S. A, Lomov, S. V., Harwood, R. J., Cassidy, T., Farrer, C.** The Simulation of the Geometry of Two-component Yarns. Part I: The Mechanics of Strand Compression: Simulating Yarn Cross-section Shape *J. Text. Inst.* 88 (2) 1997: pp. 118 – 131.
8. **Zareckas, S. G. S.** Mechanical Properties of Yarns at the Twisting. Moscow, Light Industry, 1979 (in Russian).
9. **Binkevičius, A.** Predicting of Physical and Mechanical Properties of Union Yarns *Dr. Sc. Thesis*, Kaunas Polytechnical Institute, Kaunas, Lithuania, 1985 (in Russian).
10. **Zemlekov, V. I., Popov, L. N.** Packing of Cross-section of Filament Yarn during Axial Tension *Technol. Textil. Industr.* 3 1988: pp. 11 – 14 (in Russian).
11. **Petrulis, D.** Peculiarities of Packing Fraction of Open-packed Yarn Model *Materials Science* (*Medžiagotyra*) 1 (9) 2003: pp. 116 – 119.
12. **Hearle, J. W. S., Merchant, V. B.** Relation between Specific Volume, Count and Twist of Spun Nylon Yarns *Text. Res. J.* 33 1963: pp. 417 – 424.
13. **Morris, P. J., Merkin, J. H., Rennell, R. W.** Modelling of Yarn Properties from Fibre Properties *J. Text. Inst.* 90 (3) 1999: pp. 322 – 335.
14. **Iyer Balakrishna, K., Phatarfod, R. M.** Some Aspects of Yarn Structure *J. Text. Inst.* 56 1965: pp. 225 – 247.
15. **Gracie, P. S.** Twist Geometry and Twist Limits in Yarns and Cords *J. Text. Inst.* 51 (7) 1960: pp. 271 – 288.
16. **Hearle, J. W. S.** In **Chou, T. W., Ko, F.** (eds.) Textile Structural Composites. Moscow, Mir, 1991 (in Russian).
17. **Miravete, A.** 3-D Textile Reinforcements in Composite Materials. Woodhead Publishing Limited, 1999.