

## Impact Analysis of Geometric Characteristics and Boundary Conditions on the Stiffness of Sheet Metal Parts

Huaying WU<sup>1,2\*</sup>, Bingheng LU<sup>1,2</sup>, Cheng GUO<sup>1</sup>, Yongxin WANG<sup>1,2</sup>

<sup>1</sup> School of Mechanical Engineering, Xi'an Jiaotong University, No.28, Xianning West road, Xi'an, Shaanxi, 710049, China

<sup>2</sup> National Engineering Research Center of Rapid Manufacturing (Xi'an Rapid Manufacturing Engineering Research Co.,Ltd.), No.99, Yanxiang Road, Xi'an, Shaanxi, 710054, China

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This article discusses the traditional stiffness characterization of sheet metal, examined the impact sensitivity of general cover panel geometry to the sheet metal stiffness, and determines the main factors affecting it. The sheet metal stiffness was characterized using both material and geometry properties. Extensive study was conducted on the effects of boundary constraints to the sheet metal stiffness, along with analysis of same range center stiffness variations due to different size boundaries and positions. These research results showed that for automotive body panels, internal bulge or dent geometric feature is the most sensitive factor affecting its stiffness, as long as its height is within the range of (10–25) mm. Under unilateral constraint conditions, the sheet metal stiffness exhibits a logarithmic relationship with respect to material and geometry properties; under bilateral constraint conditions it's linear. The stiffness in the same size range is basically the same even when the boundary dimensions are different. Within the same sheet metal, the stiffness in the same size range is the minimum around the edges, and basically is the same across the internal regions. This research work provided significant insights and guidance to the optimization of the main body section design, as well as the improvement on the assembly precision of body panels.

*Keywords:* stiffness; sheet metal; boundary conditions; geometric characteristics.

### 1. INTRODUCTION

Classical mechanics defines a sheet metal when its thickness of the plate  $t$  is equal to or less than the minimum size of 1/5 of the intermediate surface [1]. Car BIW (Body-in-White) is assembled with multiple complex spatial thin-walled sheets and shells through welding. Stiffness is one of the key performance characters for cover panel, which is defined as the ability to resist external load beyond the elastic deformation limits. With the development of automobile weight reduction, ultrathin and high strength sheet metals are more widely used, thus the cover panel stiffness has received increasingly focused attention [2, 3].

Insufficient stiffness may result in poor shape precision (susceptible to deformation after assembly), generate noises and vibrations when the car is in motion, and lower comfort levels [4]. Auto body weight reduction materials could easily cause increased springback of sheet metal, which makes it crucial to optimize geometric characteristics to improve the stiffness during the main body section design. It also increases the difficulty of the body assembly precision control. Thus it's fair to say that sheet metal stiffness affects the entire process of auto body panels, from design to manufacturing.

Stefan Holmberg [5] studied four material properties affecting sheet stiffness, and concluded that stress, strain, yield stress and others all have a major impact on the stiffness. Gunnar Ekstrand [6] tested various sheet stiffness, and found that the stiffness increases with thickness (as expected). Boundary conditions have a

relatively large effect on stiffness: higher stiffness is expected when more boundary constraints are in place, and lower stiffness when more springback. Boundary conditions in this paper mean the locating and clamping methods and dimensions of sheet metal.

In general, fixture elements can be classified by functionality into locators and clamps. Cai etc. proposed the N-2-1 fixture principle for compliant sheet metal parts. They concluded that in order to locate and support compliant sheet metal parts, it is necessary to provide more than three locators in the primary plane due to part flexibility [7]. This indicates that boundary constraint conditions will affect stiffness of sheet metal. Fixture layout has been researched extensively. A series of remarkable achievements on boundary conditions for improving assembly quality and efficiency have been got by complex algorithm optimization and program [8–12]. But fundamental relationship between stiffness and complex boundary conditions is studied little.

Xiao Jie et.al. [13] studied the relationships between BIW stiffness and auto body panel geometric properties, and discovered their relatively strong nonlinear relationships. These case studies showed that sheet stiffness is not only dependent on close relationships with the material properties, but also is affected directly by the sheet geometry characteristics and boundary conditions. However there still lacks accurate mathematical models that combine both material properties and geometric characteristics.

As we all know, if the stiffness of thin sheet increases significantly after additions of other characteristics, it's an effect not only caused by stress, strain, hardening and so on, but also by geometric characteristics and boundary conditions of the sheet. At locations of low stiffness on the

\*Corresponding author. Tel.: +86-029-83399061-605; fax.: +86-029-83399133. E-mail address: [wuhy@mail.xjtu.edu.cn](mailto:wuhy@mail.xjtu.edu.cn) (H. Wu)

cover panel, flexibility is larger and thus can accommodate larger assembly errors, so it may be desirable for process since this allows the compensation for error, or relaxation of tolerance limits. On the other hand, areas with higher stiffness will experience difficult adjustment of part flexibility, whose tolerance limit must be kept tight in order to ensure proper assembly.

Therefore, this paper focuses on studying the effect to the stiffness of steel sheet geometry dimensions and boundary conditions, analyzing the sensitivity of common cover panel geometric features on stiffness. The effects of geometry dimensions on stiffness are analyzed by methods of simulation and experiments. Accurate stiffness characterization model is built taking into consideration of material properties. The effects of sheet different boundary range and location on the stiffness are also explored.

## 2. STIFFNESS

The conventional definition for symbols is introduced before the stiffness is characterized:  $K$  – stiffness (N/mm);  $F$  – load force (N);  $\delta$  – displacement (mm);  $E$  – elastic modulus (N/mm<sup>2</sup>);  $I$  – moment of inertia (mm<sup>4</sup>);  $\nu$  – the Poisson's ratio;  $t$  – sheet thickness (mm);  $C$  – constant with a value about 0.2 ~ 0.22;  $R_1, R_2$  – radius along the hyperbolic primary and secondary directions (mm).

1) General definition [14]:

Stiffness is defined as the ability to resist deformation of the component; in other words, the force required to generate unit displacement:

$$K = \frac{F}{\delta} \quad (1)$$

2) Material mechanics [14]:

When material mechanics is brought to the discussion of stiffness, it usually includes both bending stiffness and torsion stiffness. In this paper, we primarily deal with the bending stiffness, which is defined as:

$$K = EI \quad (2)$$

3) Elastic mechanics, theory of plates and shells characterization [15, 16]:

Bending stiffness of sheet is defined by elastic mechanics, theory of plates and shells as follows:

$$K = \frac{Et^3}{12(1-\nu^2)} \quad (3)$$

4) Literature research [2, 6]:

Stiffness of common hyperbolic geometry for outer cover panel was characterized in the literature as:

$$K = \frac{F}{\delta}, F = C2\pi E \frac{t^{2.2}}{\sqrt{R_1 R_2}} \delta^{0.8} \quad (4)$$

The units of stiffness from equations (1)–(4) are N/mm, N·m<sup>2</sup>, N·m and N/mm. It's obvious that stiffness can be evaluated on different characteristics under different scenarios. Equation (1) is the general expression of stiffness. Equation (2) is mainly used to characterize the stiffness of structure member. Equation (3) mainly refers stiffness of plates and shells, it includes some material properties while information of boundary condition and geometric characteristic are not included. Equation (4)

mainly refers to stiffness of common hyperbolic geometry for outer cover panel, and it still based on equation (1). So the stiffness in this paper is characterized by equation (1), while at the same time taking into consideration of elements from equations (2) and (3).

## 3. SENSITIVITY OF STIFFNESS DUE TO COVER PANEL BOUNDARY AND INTERNAL GEOMETRY FEATURES

Cover panel has complex geometry features, which include curved surface, flanging and internal bulge or dent areas, etc., and the dimensions vary widely. Based upon these characteristics, a typical cover panel containing flanging and internal structure (see Fig. 1) is used to research the sensitivity of geometry features on stiffness by analyzing the sheet offset. This design will disregard the influences caused by the changes of material thickness.

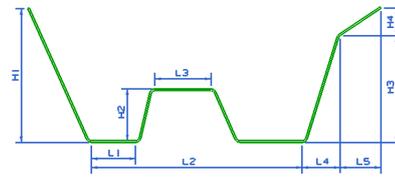


Fig. 1. A typical cover panel cross sectional view

Orthogonal test method is a scientific calculation based on professional and technical knowledge, empirical research, probability theory and mathematical statistics. It sets up experiment scheme by using standardized orthogonal table to lower the number of experiments, and reduce the test cycle. The test results are analyzed to efficiently and effectively determine the optimization scheme [17]. Nine independent dimensions of Fig. 1 are considered as factors. When H2 is positive, the characterization is bulge; dent when H2 is negative.

Based on the common dimensions of a cover panel, they are set as 8 levels (unit of mm). With the stiffness value as the goal, nine-factor and eight-level orthogonal test table is set up to analyze the effect sensitivity of feature dimensions on stiffness, and derive the relationship between geometry features and stiffness.  $L_{64}(8^9)$  is chosen after reviewing the standard orthogonal table. Orthogonal test table head is designed as Table 1.

An orthogonal test simulation model is built as illustrated in Fig. 2, with the testing goal of the stiffness (acquired by equation (1)) along Z direction where there is a load applied.

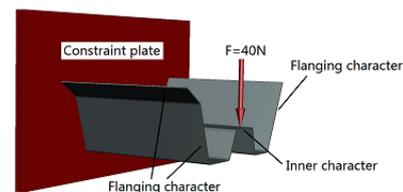


Fig. 2. Simulation model of orthogonal test

The simulation is run on the finite element analysis software ANSYS.  $E$  and  $\nu$  are set as 2.068E5 and 0.3 respectively. Shell63 element is used. Experiment scheme and results are shown in Table 2. The order of influence among these factors is determined by R. The factor level

**Table 1.** Orthogonal test table head

Levels	Factors								
	L	L2	L3	L4	L5	H1	H2	H3	H4
1	30	300	30	30	20	0	-20	0	0
2	70	500	60	70	40	5	-15	5	10
3	110	700	90	110	60	10	-10	10	20
4	150	900	120	150	80	15	-5	15	30
5	190	1100	150	190	100	25	0	25	40
6	230	1300	180	230	120	35	5	35	50
7	270	1500	210	270	140	80	10	80	60
8	310	1700	240	310	160	130	15	130	70

**Table 2.** Test program and results

Experimental number	Column number									K, (N/mm)
	L1	L2	L3	L4	L5	H1	H2	H3	H4	
1	1	1	1	1	1	1	1	1	1	888.71
2	1	2	2	2	2	2	2	2	2	864.34
3	1	3	3	3	3	3	3	3	3	257.17
...	...	...	...	...	...	...	...	...	...	...
56	7	8	2	1	5	3	6	7	4	126.61
...	...	...	...	...	...	...	...	...	...	...
63	8	7	2	6	4	5	8	3	2	1015.36
64	8	8	1	5	3	6	7	4	1	301.39
K1	3253.82	4264.51	2934.66	3546.05	3728.39	3862.23	7362.74	3943.37	3475.98	
K2	4571.95	3641.47	5139.46	4083.47	3617.23	3602.62	5873.93	3145.93	5246.81	
K3	4053.66	3546.11	4932.61	3329.65	4213.35	2745.78	3410.01	3267.83	3334.21	
K4	3018.63	3878.29	4139.10	4184.49	3832.51	3566.90	1789.57	3999.02	2899.43	
K5	4273.18	3195.46	4118.84	3031.66	3997.12	4380.23	320.51	3282.61	4804.94	
K6	3676.29	4316.83	3447.09	4305.33	3628.62	3234.70	1425.85	3800.54	3929.86	
K7	3879.65	3662.31	2871.48	3511.31	4083.57	4616.35	3720.99	4176.94	3335.88	
K8	3248.85	3471.05	2392.79	3984.07	2875.24	3967.22	6072.42	4359.79	2948.92	
k1	406.73	533.06	366.83	443.26	466.05	482.78	920.34	492.92	434.50	
k2	571.49	455.18	642.43	510.43	452.15	450.33	734.24	393.24	655.85	
k3	506.71	443.26	616.58	416.21	526.67	343.22	426.25	408.48	416.78	
k4	377.33	484.79	517.39	523.06	479.06	445.86	223.70	499.88	362.43	
k5	534.15	399.43	514.85	378.96	499.64	547.53	40.06	410.33	600.62	
k6	459.54	539.60	430.89	538.17	453.58	404.34	178.23	475.07	491.23	
k7	484.96	457.79	358.94	438.91	510.45	577.04	465.12	522.12	416.99	
k8	406.11	433.88	299.10	498.01	359.40	495.90	759.05	544.97	368.61	
Range (R)	194.17	140.17	343.33	159.21	167.26	233.82	880.28	151.73	293.42	
The sequence of factors	H2 > L3 > H4 > H1 > L1 > L5 > L4 > H3 > L2									

Note:  $K_i$  is the indicator sum of each factor in  $i$  level,  $k_i = K_i/8$ ; range  $R = k_{i_{max}} - k_{i_{min}}$ .

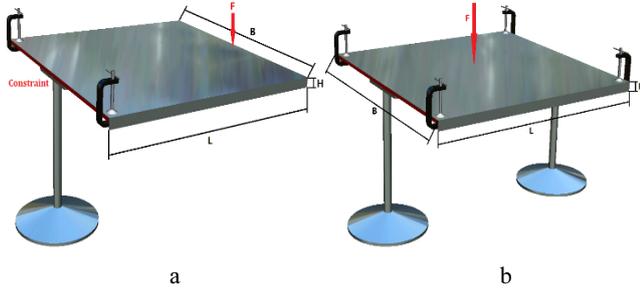
value change has more influence on test index when its  $R$  is larger, so that factor is more important. The analysis results show that the factors affecting order is  $H2 > L3 > H4 > H1 > L1 > L5 > L4 > H3 > L2$  in this model. This indicates that internal structural features of the cover panel are the most important factors affecting stiffness. Thus adding features on inner sheet metal is the most immediate and effective approach to improve the stiffness of cover panel. Flanging structure in cover panel edge can effectively improve its stiffness, too, when it's used to connect parts of spot welding assembly.

#### 4. INFLUENCE OF FLANGING STRUCTURE ON SHEET STIFFNESS

##### 4.1. Influence of dimension changes on sheet stiffness

The most sensitive feature of Fig. 2,  $L$ , is extracted and set as sheet length.  $B$  is width and  $H$  is height. The principal moment of inertia is used to quantify sheet shape, and its influence on stiffness is studied. When  $B$  and  $L$  values are small,  $H$  can be regarded as the internal structure height of cover panel; when  $B$  and  $L$  are larger,  $H$

can be regarded as the edge flanging height of cover panel. Drawing in Fig. 3, a, represents the condition of unilateral constraint with a load along edge midpoints, and Fig. 3, b, bilateral constraint with a load in geometric center. The influence of different dimension changes on stiffness is analyzed.

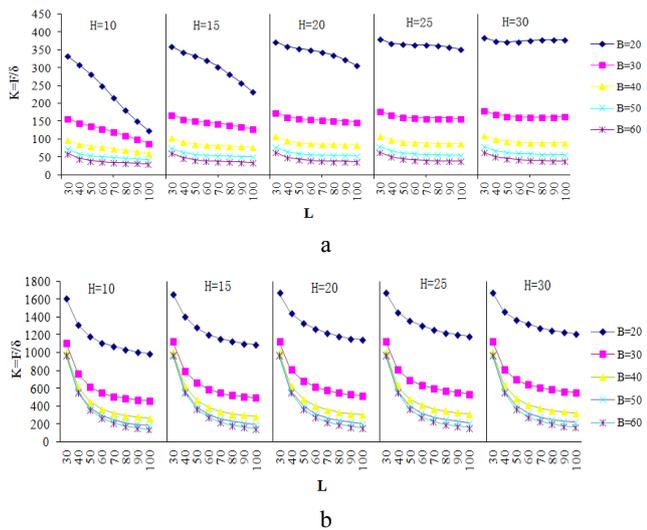


**Fig. 3.** Stiffness analyzing model of  $L \times B \times H$ : a – unilateral constraint; b – bilateral constraint

According to the general internal geometric feature size, these free combination dimensions are used (total combination amount is 200, unit: mm):

$$L \times B \times H = \begin{Bmatrix} 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \\ 90 \\ 100 \end{Bmatrix} \times \begin{Bmatrix} 20 \\ 30 \\ 40 \\ 50 \\ 60 \end{Bmatrix} \times \begin{Bmatrix} 10 \\ 15 \\ 20 \\ 25 \\ 30 \end{Bmatrix}. \quad (5)$$

Suppose the sheet thickness is 0.7 mm. The simulation parameters are from the orthogonal test parameters. The simulation results on stiffness values (Fig. 4) displayed strong periodical distribution characteristics.



**Fig. 4.** Relationship between computational stiffness and  $F/\delta$ ,  $L$ ,  $B$  and  $H$ : a – unilateral constraint; b – bilateral constraint

Under the same heights, Fig. 4, a and b, show that the trend of stiffness changes is similar to that of  $B$  and  $L$  variation: when  $B$  and  $L$  values are increased, stiffness drops, which can be manifested by the dramatic changes in

Fig. 4, b; under different heights, Fig. 4, a, shows that  $L$  and  $B$  have slightly different effects on the trend of stiffness variation, while Fig. 4, b, indicates that the influence on the stiffness by  $L$  and  $B$  values is almost negligible.

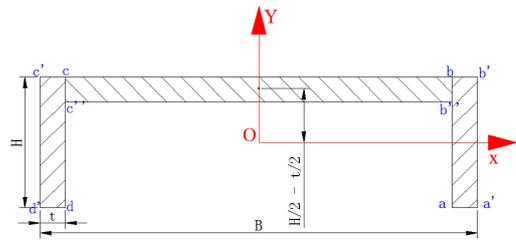
Moment of inertia refers to the integral cross-section of each element area, and the product of a specified axis of each element to the cross section from the quadratic. It's a measure of an object's resistance to any change in its state of rotation. The moment of inertia for  $X$  axis is defined as:  $I_x = \int Ay^2 dA$ . The center of 'a'b'c'd' (see Fig. 5) is regarded as the coordinate origin  $O$  to establish one coordinate system. The moment of inertia is calculated by dividing the sheet metal section into three parts (equation (6)).

$$I_{d'dcc'} = I_{aa'b'b} = \frac{t}{12} \cdot H^3$$

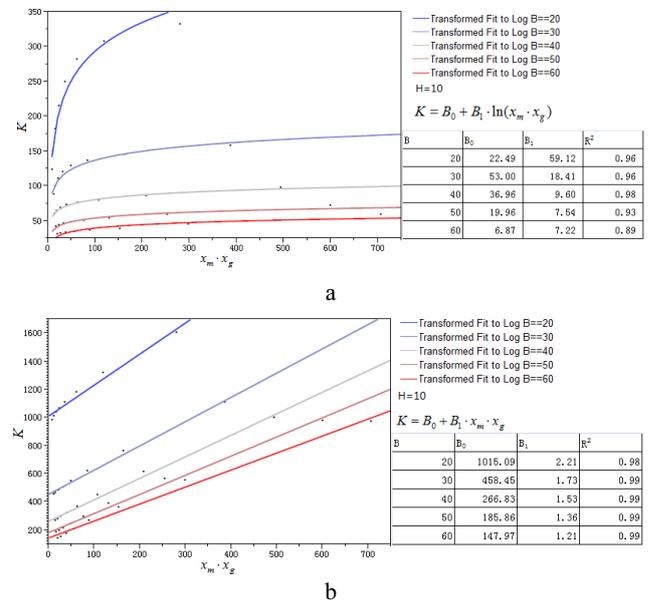
$$I_{c''b''bc} = \frac{(B - 2t)}{12} \cdot t^3 + \left(\frac{H}{2} - \frac{t}{2}\right)^2 \cdot (B - 2t) \cdot t$$

$$I_{section} = I_{d'dcc'} + I_{c''b''bc} + I_{aa'b'b}$$

$$= \frac{(B - 2t)t^3}{12} + \frac{t(B - 2t)(H - t)^2}{4} + \frac{tH^3}{6} \quad (6)$$



**Fig. 5.** Moment of inertia for geometric cross section



**Fig. 6.** The relationship between  $K$  and when  $H = 10$ : a – unilateral constraint; b – bilateral constraint

Material characterization is expressed as  $x_m = E/12(1 - \nu^2)$ , geometry characterization is given by  $x_g = I/L^3$ , and  $K$  is studied by varying  $x_m$  and  $x_g$ . The relationship is obtained by fitting the values of  $K$ ,  $x_m$  and

$x_g$  are illustrated in Fig. 4. The trends are almost the same with different height values. When the constraint is unilateral, the relationship is logarithmic:

$$K = B_0 + B_1 \cdot \ln(x_m \cdot x_g). \quad (7)$$

When the constraint is bilateral, the relationship is linear:

$$K = B_0 + B_1 \cdot x_m \cdot x_g. \quad (8)$$

$B_0$  and  $B_1$  are coefficients which correspond to different numeric values depending on B and H. For example, when  $H = 10$  and B varies from 20 to 60, the relationship of  $K$  and  $B$  is shown in Fig. 6. Different B corresponds to different  $B_0$  and  $B_1$ . And all curve fitting degree indexes ( $R^2$  value) indicate a high fitting accuracy under different B value.

## 4.2. Experiments and simulations on sheet stiffness

A plate stiffness of material Q235 which  $L \times B \times t = 200 \times 200 \times 0.7$  (mm) is tested. The flanging height H is taken as 0, 10, 15, 20, 25, 30 (mm) respectively. The plate is clamped in supported plate whose dimension is  $200 \times 20 \times 5$  (mm). WDW-50 microcomputer control electronic universal testing machine is used as the test facility. The plate stiffness is measured using a hemispherical head punch of diameter is 12.7 mm [18]. Stiffness experimental data of unilateral constraint and bilateral constraint (see Fig. 7) are compared with finite element simulation results (see Fig. 8), and the conclusions can be acquired as follows:

1) The experimental data and simulation results are almost consistent, it proved that the simulation results are credible.

2) Under unilateral constraint, the stiffness of sheet increasing rapidly with increasing the flanging height.

3) The stiffness of sheet increasing with the increasing flanging height no matter under what kind of constraint it is. The increasing trend slowing after the flanging height is increased to a certain extent. For the  $200 \times 200$  (mm) sheet, stiffness increasing obviously slow down after flanging height is increased to (10 ~ 15) mm.

4) Bilateral constraint reduces the effect on the stiffness caused by flanging height change.

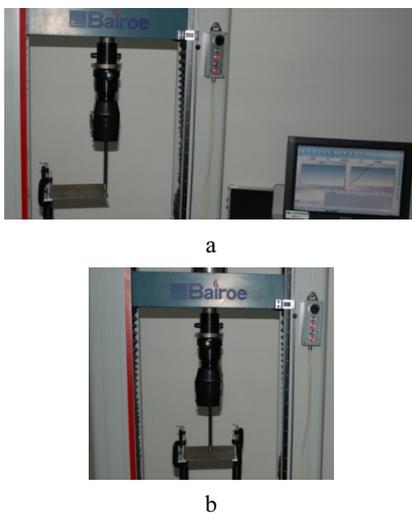


Fig. 7. Stiffness experiment: a – unilateral constraint; b – bilateral constraint

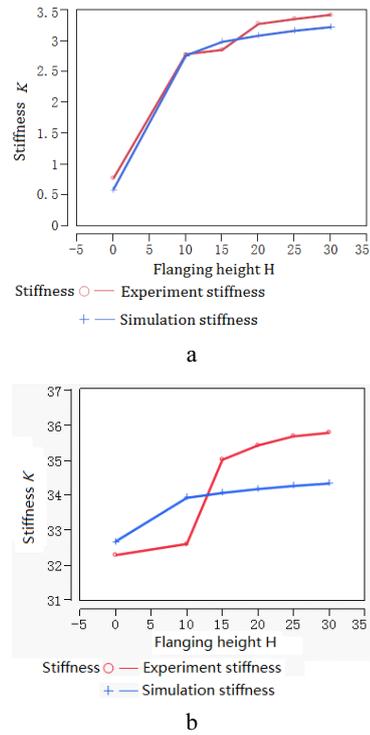


Fig. 8. Stiffness comparison in different flanging height of  $200 \times 200$  (mm) sheet: a – unilateral constraint; b – bilateral constraint

## 5. IMPACT OF BOUNDARY CONDITIONS TO THE SHEET STIFFNESS

### 5.1. Comparison of stiffness in the same size range under different dimension boundaries

Sheet metal exhibits different stiffness under different boundary constraints. When a larger sized sheet is clamped at multiple locations, it becomes equivalent to the case as if the sheet is divided into many small ranges, and thus causing redistribution of stiffness. Based on this principle, it's worthwhile to study the relationship between individual size ranges with respect to the entire sheet to be divided, as this can provide optimization base for larger sized sheet panel clamped and fixed in the assembly process. Fig. 9 shows an example of a simulation study (units in millimeters): the sheets with a thickness of 0.7, length and width of  $800 \times 800$ ,  $400 \times 400$ ,  $200 \times 200$ ,  $100 \times 100$  respectively, are constrained along the boundaries. The center stiffness of  $100 \times 100$  range is simulated and analyzed.

From the results (Fig. 10) we can conclude that the center stiffness is the lowest in  $100 \times 100$  size range, when it is equal to boundary dimension (this sheet is then defined as an independent single sheet. When the boundary dimension of a sheet is just same as the size range which is needed to be analyzed, the sheet is defined as independent single sheet for this size range. For example, if the stiffness of size range  $100 \times 100$  is needed to be analyzed, a sheet with boundary dimension of  $100 \times 100$  is called an independent single sheet for it.). When the boundary dimension is increased, the center stiffness of  $100 \times 100$  size range increases about 1.25 times of an independent single sheet of  $100 \times 100$ . In boundary dimensions of

200 × 200, 400 × 400 and 800 × 800, the center stiffness in 100 × 100 range is almost the same. Thus it can be concluded that under different boundary dimensions, the stiffness is equal in the same range.

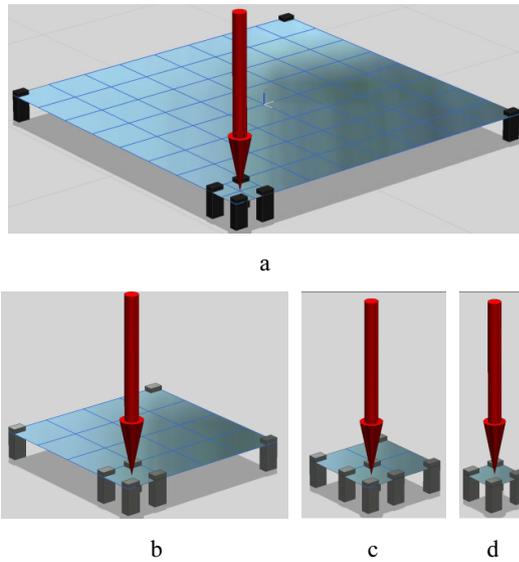


Fig. 9. Simulation model of different boundary conditions for stiffness calculations: a – 800; b – 400; c – 200; d – 100

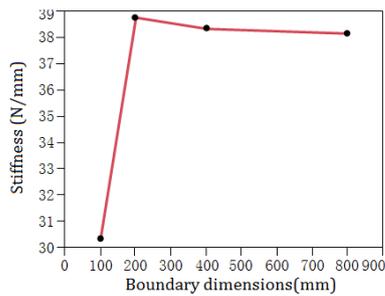


Fig. 10. Center stiffness of 100 × 100 range under different boundary dimensions

## 5.2. Comparison of stiffness in different positions with the same range

A sheet of 800 × 800 is divided into 50 × 50 unit cells, and the center stiffness  $k_{ij}$  of 100 × 100 range from all cells is simulated in Fig. 11, a, with  $i = \{-350, -300, -250, \dots, 250, 300, 350\}$  and  $j = \{50, 100, 150, \dots, 650, 700, 750\}$ . Analysis results (Fig. 11, b) show that in 800 × 800 range, stiffness on peripheral edges of 100 × 100 range is relatively low, with its minimum at the four corners of the sheet; while in the interior of the 100 × 100 range, the stiffness basically is the same.

With the calculations from Fig. 10, we can propose that the center stiffness in 100 × 100 range is the lowest when it is equal to boundary dimensions; for large sized sheet, stiffness is redistributed when it is constrained and fixed at various positions. Edge stiffness of the same range is about N times of an independent single sheet. Multiple dimensions are analyzed and the value of N is found to be between 1.0 and 2.0. The internal stiffness value in the same range has little difference, which is about 1.5 times of the independent single sheet.

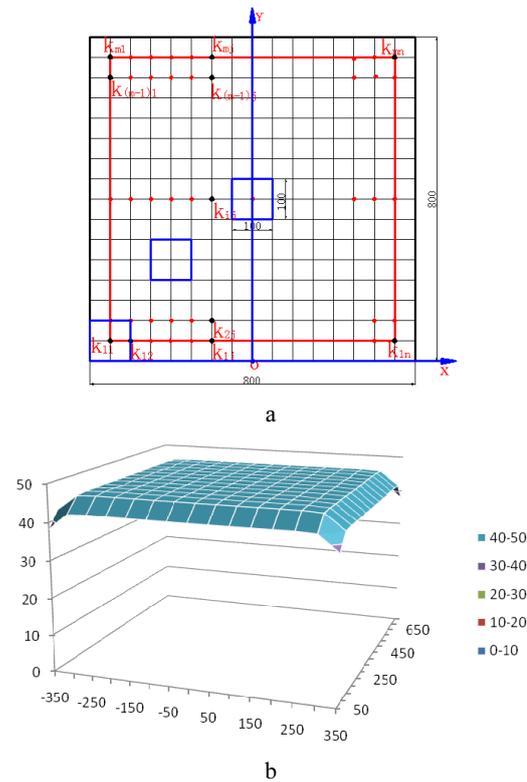


Fig. 11. Center stiffness of 100 × 100 range in different positions within 800 × 800: a – stiffness value location sketch map; b – analysis results

## 6. CONCLUSIONS

The following conclusions can be obtained from this study:

1) The bulge or dent geometry character is the most sensitive effect factor on cover panel stiffness.

2) The variation of bulge or dent structure dimensions has important influence on stiffness. Stiffness is integrated characterized by material properties and geometric properties by comparing classical mechanics. Under unilateral constraint, the relationship of and is logarithmic; under bilateral constraint, the relationship is linearity. Length and width have non-linearity inverse relationship with stiffness. Bigger height, bigger stiffness, while in certain length and width, continue increasing height has little effect on stiffness increasing. For 1 × 0.6 (meter) range sheet metal, the proper height which can improve stiffness effectively would be (10 ~ 25) mm after researching the geometric size in this range.

3) Constraints more, stiffness higher. A big size sheet metal in multiple constraint can be regarded as which is divided into a plurality of small size range. The stiffness is a combination of these small size range. The stiffness in the same range are almost the same although the boundary dimensions are different, and they are all about 1 ~ 2 times when the range is equal to its boundary dimensions. Within the same sheet, the stiffness in edge is the smallest and interior is consistent.

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